

1. Imagine a spin-1/2 system prepared in the state:

$$|\psi\rangle = \sqrt{\frac{2}{5}}|+\rangle + i\sqrt{\frac{3}{5}}|-\rangle$$

Use a completeness relation to write this state in the S_x basis.

Solution

$$\begin{aligned} |\psi\rangle &= (1)\left(\sqrt{\frac{2}{5}}|+\rangle + i\sqrt{\frac{3}{5}}|-\rangle\right) \\ &= \left(|+\rangle_x \langle +| + |-\rangle_x \langle -|\right)\left(\sqrt{\frac{2}{5}}|+\rangle + i\sqrt{\frac{3}{5}}|-\rangle\right) \\ &= \sqrt{\frac{2}{5}}|+\rangle_x \langle +| + \frac{1}{\sqrt{2}}\sqrt{\frac{3}{5}}|+\rangle_x \langle +| + i\sqrt{\frac{3}{5}}|+\rangle_x \langle -| + \frac{1}{\sqrt{2}}\sqrt{\frac{3}{5}}|-\rangle_x \langle +| \\ &\quad + \sqrt{\frac{2}{5}}|-\rangle_x \langle -| + \frac{1}{\sqrt{2}}\sqrt{\frac{3}{5}}|-\rangle_x \langle -| - \frac{1}{\sqrt{2}}\sqrt{\frac{3}{5}}|-\rangle_x \langle +| \\ &= \sqrt{\frac{1}{5}}|+\rangle_x + i\sqrt{\frac{3}{10}}|+\rangle_x + \sqrt{\frac{1}{5}}|-\rangle_x - i\sqrt{\frac{3}{10}}|-\rangle_x \\ &= \left(\sqrt{\frac{1}{5}} + i\sqrt{\frac{3}{10}}\right)|+\rangle_x + \left(\sqrt{\frac{1}{5}} - i\sqrt{\frac{3}{10}}\right)|-\rangle_x \end{aligned}$$

2. Use a completeness relation to write $|-\rangle$ in the S_y basis.

Solution

$$\begin{aligned} |-\rangle &= (1)|-\rangle \\ &= \left(|+\rangle_y \langle +| + |-\rangle_y \langle -|\right)|-\rangle \\ &= |+\rangle_y \langle +| + \frac{-i}{\sqrt{2}}|+\rangle_y \langle -| + |-\rangle_y \langle -| + \frac{i}{\sqrt{2}}|-\rangle_y \langle +| \\ &= \frac{-i}{\sqrt{2}}|+\rangle_y + \frac{i}{\sqrt{2}}|+\rangle_y \\ &= -i\left(\frac{1}{\sqrt{2}}|+\rangle_y - \frac{1}{\sqrt{2}}|+\rangle_y\right) \end{aligned}$$