$$|v\rangle \doteq \frac{1}{\sqrt{74}}\begin{pmatrix}7\\5i\end{pmatrix}$$

$$|+\rangle \doteq \begin{pmatrix}1\\0\end{pmatrix} \qquad |-\rangle \doteq \begin{pmatrix}0\\1\end{pmatrix}$$

1. You can expand a two-component vector with complex entries in terms of the standard basis, $|+\rangle$ and $|-\rangle$. Find the coefficients v_+ and v_- for the vector $|v\rangle$ in the expression

$$|v\rangle = v_+\,|+\rangle + v_-\,|-\rangle$$

Solution

$$v_+ = \langle +|v\rangle \tag{1}$$
$$= \begin{pmatrix}1 & 0\end{pmatrix}\frac{1}{\sqrt{74}}\begin{pmatrix}7\\5i\end{pmatrix} \tag{2}$$
$$= \frac{7}{\sqrt{74}} \tag{3}$$

$$v_- = \langle -|v\rangle \tag{4}$$
$$= \begin{pmatrix}0 & 1\end{pmatrix}\frac{1}{\sqrt{74}}\begin{pmatrix}7\\5i\end{pmatrix} \tag{5}$$
$$= \frac{5i}{\sqrt{74}} \tag{6}$$

2. Try a different basis:

$$|+\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} \qquad |-\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

Find:

$$|v\rangle = v_{+y}\,|+\rangle_y + v_{-y}\,|-\rangle_y$$

Solution

$$v_{+y} = {}_y\langle +|v\rangle \tag{7}$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\end{pmatrix}\frac{1}{\sqrt{74}}\begin{pmatrix}7\\5i\end{pmatrix} \tag{8}$$
$$= \frac{12}{\sqrt{128}} \tag{9}$$

$$
\begin{aligned}
v_{-y} &= {}_y\langle -|v\rangle && (10)\\
&= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & i\end{pmatrix}\frac{1}{\sqrt{74}}\begin{pmatrix}7\\ 5i\end{pmatrix} && (11)\\
&= \frac{2}{\sqrt{128}} && (12)
\end{aligned}
$$