

Cylindrical Coordinates:

Find the general form for $d\vec{r}$ in cylindrical coordinates by determining $d\vec{r}$ along the specific paths below.

- Path 1 from (s, ϕ, z) to $(s + ds, \phi, z)$:

$$d\vec{r} =$$

- Path 2 from (s, ϕ, z) to $(s, \phi, z + dz)$:

$$d\vec{r} =$$

- Path 3 from (s, ϕ, z) to $(s, \phi + d\phi, z)$:

$$d\vec{r} =$$

If all three coordinates are allowed to change simultaneously, by an infinitesimal amount, we could write this $d\vec{r}$ for any path as:

$$d\vec{r} =$$

This is the general line element in cylindrical coordinates.

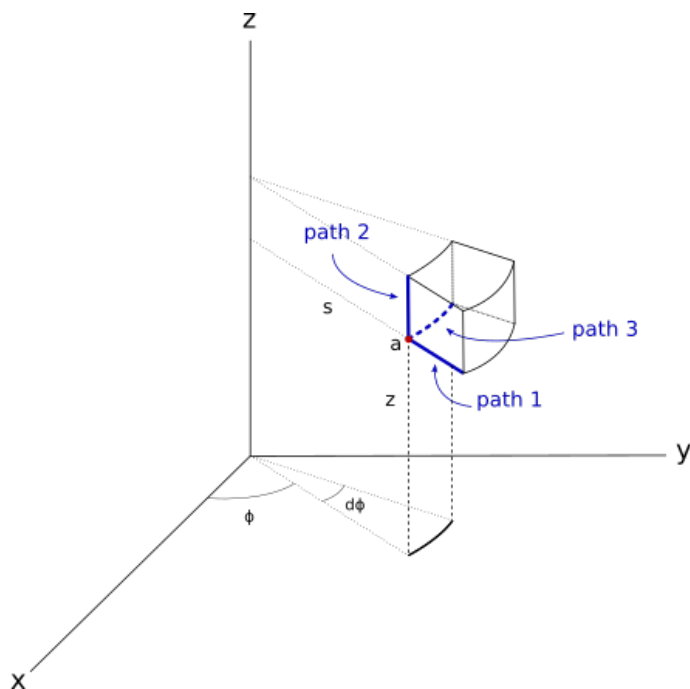


Figure 1: Figure 1: $d\vec{r}$ in cylindrical coordinates

Solution

- Path 1:

$$d\vec{r} = ds \hat{s}$$

- Path 2:

$$d\vec{r} = dz \hat{z}$$

- Path 3:

$$d\vec{r} = s d\phi \hat{\phi}$$

$d\vec{r}$ is an infinitesimal length vector whose magnitude is a length. On path 3, it is necessary to use the arclength formula to turn the infinitesimal angle $d\phi$ into an appropriate length.

If all three coordinates are allowed to change simultaneously, by an infinitesimal amount, we could write this $d\vec{r}$ for any path as:

$$d\vec{r} = ds \hat{s} + s d\phi \hat{\phi} + dz \hat{z}$$

This is the general line element in cylindrical coordinates.

Spherical Coordinates:

Find the general form for $d\vec{r}$ in spherical coordinates by determining $d\vec{r}$ along the specific paths below.

- Path 1 from (r, θ, ϕ) to $(r + dr, \theta, \phi)$:

$$d\vec{r} =$$

- Path 2 from (r, θ, ϕ) to $(r, \theta + d\theta, \phi)$:

$$d\vec{r} =$$

- Path 3 from (r, θ, ϕ) to $(r, \theta, \phi + d\phi)$: (Be careful, this is a tricky one!)

$$d\vec{r} =$$

If all three coordinates are allowed to change simultaneously, by an infinitesimal amount, we could write this $d\vec{r}$ for any path as:

$$d\vec{r} =$$

This is the general line element in spherical coordinates.

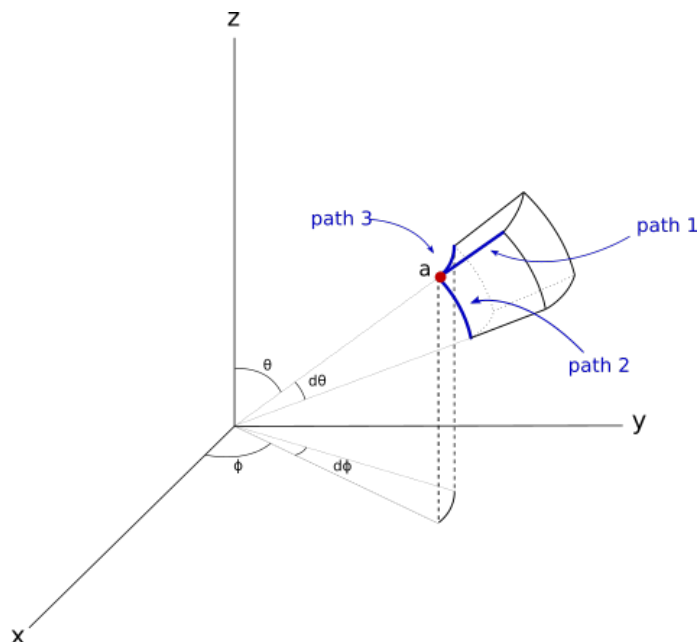


Figure 2: Figure 2: $d\vec{r}$ in spherical coordinates

Solution

- Path 1:

$$d\vec{r} = dr \hat{r}$$

- Path 2:

$$d\vec{r} = r d\theta \hat{\theta}$$

- Path 3:

$$d\vec{r} = r \sin \theta d\phi \hat{\phi}$$

If all three coordinates are allowed to change simultaneously, by an infinitesimal amount, we could write this $d\vec{r}$ for any path as:

$$d\vec{r} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$$

This is the general line element in spherical coordinates.