

Estimate the optical depth for infrared light with wavelength $15 \mu\text{m}$ when it travels through our atmosphere at standard temperature and pressure *STP*. The number density of air molecules at STP can be found from $pV = Nk_B T$.

Our air is 0.041% CO_2 (410 ppm).

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Figure 1: Absorption cross sections σ_a . The graph is from *Fundamentals of Atmospheric Radiation* by Craig Bohren, a highly recommended book.

Solution We need to calculate the optical depth $= \frac{1}{n\sigma_a}$. From the plot

$$\sigma_a = 10^{-11} \mu\text{m}^2 = 10^{-23} \text{ m}^2 \quad (1)$$

n is the number of CO_2 molecules per unit volume.

We know that there are 410 CO_2 molecules for every 10^6 air molecules (410 ppm). This is equivalent to 0.041%.

How many air molecules in 1 m^3 ?

$$pV = Nk_B T \quad (2)$$

$$\frac{N}{V} = \frac{p}{k_B T} = \frac{10^5 \text{ N/m}^2}{[1.4 \times 10^{-23} \text{ J/K}][300 \text{ K}]} \quad (3)$$

$$= \frac{1}{4.2} \times 10^{26} \text{ m}^{-3} \quad (4)$$

$$= 2.4 \times 10^{25} \text{ m}^{-3} \quad (5)$$

The next step is to find the number of CO_2 molecules per m^3 from the fraction of 2.4×10^{25} .

$$N_{\text{CO}_2} = [4.1 \times 10^{-4}][2.4 \times 10^{25}] \text{ m}^{-3} \quad (6)$$

$$= 10^{22} \text{ m}^{-3} \quad (7)$$

Therefore the optical depth is

$$L_{\text{opt}} = \frac{1}{n\sigma} \approx \frac{1}{[10^{22} \text{ m}^{-3}][10^{-23} \text{ m}^2]} \quad (8)$$

$$= 10 \text{ m} \quad (9)$$

Sensemaking 10 m is much less than the thickness of the atmosphere (the atmosphere is 10 km). The light with wavelength $15 \mu\text{m}$ will not go straight up into outer space. The light will bounce around, distributing its energy into the internal vibrations of air molecules.