

## 1 Fourier Transform of a Gaussian

Consider the Gaussian wave function  $f(x) = Ne^{-x^2/2\sigma^2}$ .

- Find the normalization constant  $N$ . Write a sentence describing the physical meaning of normalizing. (The identity  $\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$  may prove helpful.)
- Find the Fourier transform of  $f(x)$  by hand. You will need to “complete the square”. Sense-making: Discuss how changing the constant  $\sigma$  changes the shape of both  $f(x)$  and its Fourier transform.
- Show that the Fourier transform of  $f(x)$  is also normalized. (This is true for any function and is known as Parseval’s identity.) Write a sentence describing the physical meaning of normalizing in this case.

## 2 Fourier Transform of Cosine and Sine

- Find the Fourier transforms of  $f(x) = \cos kx$  and  $g(x) = \sin kx$ .
- Find the Fourier transform of  $g(x)$  using the formula for the Fourier transform of a derivative and your result for the Fourier transform of  $f(x)$ . Compare with your previous answer.
- In quantum mechanics, the Fourier transform is the set of coefficients in the expansion of a quantum state in terms of plane waves, i.e. the function  $\tilde{f}(k)$  is a continuous histogram of how much each functions  $e^{ikx}$  contributes to the quantum state. What does the Fourier transform of the function  $\cos kx$  tell you about which plane waves make up this quantum state? Write a sentence or two about how this makes sense.

## 3 Fourier Transform of a Triangle

Consider a quantum mechanical wave packet shaped like a triangle:

$$f(x) = \begin{cases} \sqrt{\frac{3}{2\epsilon^3}}(x + \epsilon), & -\epsilon < x < 0 \\ -\sqrt{\frac{3}{2\epsilon^3}}(x - \epsilon), & 0 < x < \epsilon \\ 0, & \text{otherwise} \end{cases}$$

- Show that the wave packet is normalized.
- For three different values of  $\epsilon$ , plot both the wave packet. (All three plots should be on the same axes.)
- Find the Fourier Transform of the wave packet by hand. You may use technology of your choice to evaluate integrals, but do not use any built in Fourier Transform packages.

- (d) For three different values of  $\epsilon$ , plot both the Fourier Transform of the wave packet. (All three plots should be on the same axes.)
- (e) As you change the value of  $\epsilon$  so that the packet gets narrower and taller, what happens to the shape of the Fourier Transform?
- (f) Show that the Fourier transform is also (norm-squared) normalized.