1 4 Coupled Masses

Consider a system of four masses coupled by springs of spring constant k to form a ring. You should consider this as a 1D system with periodic boundary condition. Initially we hold three of the masses at equilibrium location, and displace the other one by δx. At time zero, we release the displaced mass at zero velocity.

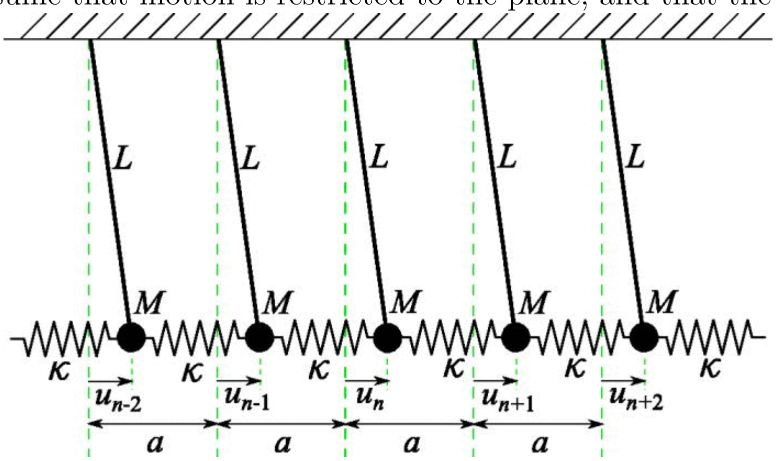
(a) If all masses are equal to m, the normal modes can be solved by the following matrix equation: $K\mathbf{x} = -\omega^2 m\mathbf{x}$. Find K, you should try to identify the pattern so that in the future you can write down the matrix K directly.

(b) What are the eigenfrequencies and normal modes of the system?

(c) What are the equations of motion for the system?

2 Chain of Coupled Pendulums

Consider an infinite periodic system of coupled **pendulums**. The length of each pendulum is L. The moving masses have mass m. A portion of the system is shown below. The springs between the masses are identical and have spring constant κ. At equilibrium the masses lie on the x-axis with a spacing a. Assume that motion is restricted to the plane, and that the amplitude of motion is small.



(a) Find the dispersion relation for small oscillations of this system.

(b) Explore the dispersion relation. This part is deliberately open ended to encourage you to ask questions yourself. Such questions could include: what are the interesting features (max freq, min freq, periodicity in the dispersion relation)? What is different about this system compared to others you have studied? Are there limiting cases as you change κ? How quantitative can you be?