1 Reflection from a Square Barrier

Consider a particle with mass m and energy E incident from the left on a square potential barrier with height $V_0 > 0$:

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 < x < a \\ 0 & a < x \end{cases}$$

This is an example of an *unbounded* system, so there is no condition on the energy eigenvalue. There are two cases, $E > V_0$ and $E < V_0$. Consider only $E > V_0$.

(a) Set up a wave incident from the left, and one transmitted to the right, so the total wave function is:

$$\psi(x) = \begin{cases} e^{ik_1x} + Ae^{-ik_1x} & x < 0 \\ Ce^{ik_3x} + De^{-ik_3x} & 0 < x < a \\ Be^{ik_2x} & a < x \end{cases}$$

Use the energy eigenvalue equation to solve for the values k_1, k_2, k_3 in terms of E and V_0.

(b) What are the boundary conditions that establish the relationship among the coefficients A, B, C, and D?

(c) The probability to observe the particle reflected is

$$r \equiv |A|^2$$

(remember we measure probabilities, and not amplitudes). Find r. Also find the probability of transmission

$$t \equiv |B|^2$$

To make the algebra easy, **you can assume that** $E = \frac{4}{3}$, $V_0 = \frac{1}{2m}(\frac{2\pi\hbar}{a})^2$.

Show that $r + t = 1$.

(d) Interpret your results, and also discuss the limiting cases:

- $V_0 = 0$,
- $E \gg V_0$,
- and $E = V_0$.

2 Coupled Oscillations: Three Masses

(2 points each)

Consider a system of three unequal masses in a row between two fixed walls. The walls and masses are all connected by unequal springs.

(a) Sketch and label a diagram for the system

(b) Write a system of coupled ODEs that represents the motion of this system.

(c) Rearrange your system of ODEs to find a matrix ODE.

(d) Impose an appropriate Ansatz for the normal modes of this system to obtain a algebraic matrix equation.

(e) Find the eigenvectors and eigenvalues of this matrix equation in the special case that the system is symmetric around the central mass and all the masses are the same.

(f) *Sensemaking*: Explain briefly what these eigenvectors tell you about the normal modes of the system and why you might expect these normal modes based on the symmetries of the system.

(g) Find the general solution for the motion of the (unforced) system in the special case that the system is symmetric around the central mass and all the masses are the same.