1 Momentum of a Free Particle

Consider a free particle whose wave function is $\psi(x) = A\sin(p_0 x/\hbar)$,

(a) Is this wave function an eigenstate of momentum?

(b) What are the possible results of a measurement of the momentum?

(c) Calculate the expectation value $\langle p \rangle$ and uncertainty Δp of momentum.

2 Plotting Dispersion Relation of a Free Particle

For a 1-D free particle, whose wave function is $\psi(x) = Ae^{ikx}$, plot its dispersion relation, namely: the energy as a function of wave vector k. Note k can be positive or negative, and the dispersion relation will come back later in the course.

3 Position and Momentum Commutation

Calculate the commutator of the position and momentum operators. Do this two ways:

(a) using the position representation of the operators

(b) using the momentum representation of the operators

4 Derivatives of the Gaussian

The normalized Gaussian function is of the form

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\, e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

(a) Find the first two derivatives of the Gaussian function, by hand.

(b) Make a table describing where the signs of the Gaussian itself and the signs of its first two derivatives are positive and negative.

(c) Use your table to describe the shape of the Gaussian function.