1 QM Boundary Condition on the First Derivative

(a) Demonstrate that the first derivative of the wave function is continuous at a point if the potential is finite at that point.

To do this, integrate the energy eigenvalue equation from $-\epsilon$ to ϵ and take the limit that $\epsilon \to 0$.

Hint 1: You should apply the fundamental theorem of calculus, which says that $\int_a^b \frac{df}{dx}\, dx = f(b) - f(a)$.

Hint 2: For a region with a *very* small width, you can consider the integration region to be a small retangle (similar to the idea behind a Reimann sum).

(b) Show that for a delta function potential of the form:

$$V(x) = \beta\delta(x)$$

this boundary condition on the first derivative of the wavefunction is:

$$\lim_{\epsilon\to 0}\left(\left.\frac{\partial\psi}{\partial x}\right|_{\epsilon} - \left.\frac{\partial\psi}{\partial x}\right|_{-\epsilon} \right) = \frac{2m\beta}{\hbar^2}\psi(0)$$

that is, the discontinuity in the derivative is proportional to the value of the wavefunction at that point.

2 Normalize the Eigenstates of the Finite Well

Consider a particle of mass m in a finite potential well:

$$V(x) = \begin{cases} V_0 & x < -a \\ 0 & -a < x < a \\ V_0 & a < x \end{cases}$$

Consider the even solutions only.

Normalize the even solutions and use the boundary conditions to help you solve to the A and D parameters in terms of q and k.