

## 1 Measurement Probabilities (Brief)

A beam of spin- $\frac{1}{2}$  particles is prepared in the initial state

$$|\psi\rangle = \sqrt{\frac{2}{5}} |+\rangle_x - \sqrt{\frac{3}{5}} |-\rangle_x$$

(Note: this state is written in the  $S_x$  basis!)

- (a) What are the possible measurement values if you measure the spin component  $S_x$ , and with what probabilities would they occur?
- (b) What are the possible measurement values if you measure the spin component  $S_z$ , and with what probabilities would they occur?

## 2 Completeness Relation Change of Basis

- (a) Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos \phi |\hat{x}\rangle + \sin \phi |\hat{y}\rangle \\ |\hat{\phi}\rangle &= -\sin \phi |\hat{x}\rangle + \cos \phi |\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle \hat{x} | \hat{s} \rangle, \quad \langle \hat{y} | \hat{s} \rangle, \quad \langle \hat{x} | \hat{\phi} \rangle, \quad \langle \hat{y} | \hat{\phi} \rangle$$

- (b) Given a vector written in the polar basis

$$|\vec{v}\rangle = a |\hat{s}\rangle + b |\hat{\phi}\rangle$$

where  $a$  and  $b$  are known.

Express  $|\vec{v}\rangle$  in the Cartesian basis,

$$|\vec{v}\rangle = c |\hat{x}\rangle + d |\hat{y}\rangle$$

by finding  $c$  and  $d$

Hint: Use the completeness relation:  $|\hat{x}\rangle \langle \hat{x}| + |\hat{y}\rangle \langle \hat{y}| = 1$

- (c) Given a quantum state written in the  $S_z$  basis,

$$|\Psi\rangle = g |+\rangle + h |-\rangle,$$

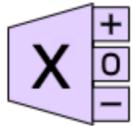
express  $|\Psi\rangle$  in the  $S_y$  basis. That is, find coefficients  $j$  and  $k$  such that

$$|\Psi\rangle = j |+\rangle_y + k |-\rangle_y.$$

### 3 Spin One Intro

The OSP Spins Laboratory simulation can also be used to explore spin-1 systems. The components of spin for these systems can be measured to be:

- $\hbar$  (corresponding to the “+” port)
- $0\hbar$  (corresponding to the “0” port)
- $-\hbar$  (corresponding to the “-” port)



To switch the simulation to a spin-1 system, find the hyperlink about halfway down the page that says “Click here to switch”.

- (a) Draw and label a diagram of an experimental setup that would allow you to prepare a set of spin-1 particles to be in the  $|1\rangle_x$  state and then measure the  $z$  component of spin for these particles.
- (b) Using the simulation, prepare a set of particles to be in the  $|1\rangle_x$  state and measure the  $x$ ,  $y$ , and  $z$  components of spin of these particles. Draw probability histograms of the results for each spin-component-direction  $S_x$ ,  $S_y$ , and  $S_z$ .