

1 Measurement Probabilities (Brief)

A beam of spin- $\frac{1}{2}$ particles is prepared in the initial state

$$|\psi\rangle = \sqrt{\frac{2}{5}} |+\rangle_x - \sqrt{\frac{3}{5}} |-\rangle_x$$

(Note: this state is written in the S_x basis!)

- What are the possible measurement values if you measure the spin component S_x , and with what probabilities would they occur?
- What are the possible measurement values if you measure the spin component S_z , and with what probabilities would they occur?

2 Completeness Relation Change of Basis

- Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos \phi |\hat{x}\rangle + \sin \phi |\hat{y}\rangle \\ |\hat{\phi}\rangle &= -\sin \phi |\hat{x}\rangle + \cos \phi |\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle \hat{x} | \hat{s} \rangle, \quad \langle \hat{y} | \hat{s} \rangle, \quad \langle \hat{x} | \hat{\phi} \rangle, \quad \langle \hat{y} | \hat{\phi} \rangle$$

- Given a vector written in the polar basis

$$|\vec{v}\rangle = a |\hat{s}\rangle + b |\hat{\phi}\rangle$$

where a and b are known.

Express $|\vec{v}\rangle$ in the Cartesian basis,

$$|\vec{v}\rangle = c |\hat{x}\rangle + d |\hat{y}\rangle$$

by finding c and d

Hint: Use the completeness relation: $|\hat{x}\rangle \langle \hat{x}| + |\hat{y}\rangle \langle \hat{y}| = 1$

- Given a quantum state written in the S_z basis,

$$|\Psi\rangle = g |+\rangle + h |-\rangle,$$

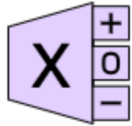
express $|\Psi\rangle$ in the S_y basis. That is, find coefficients j and k such that

$$|\Psi\rangle = j |+\rangle_y + k |-\rangle_y.$$

3 Spin One Intro

The OSP Spins Laboratory simulation can also be used to explore spin-1 systems. The components of spin for these systems can be measured to be:

- \hbar (corresponding to the “+” port)
- $0\hbar$ (corresponding to the “0” port)
- $-\hbar$ (corresponding to the “-” port)



To switch the simulation to a spin-1 system, find the hyperlink about halfway down the page that says “Click here to switch”.

- (a) Draw and label a diagram of an experimental setup that would allow you to prepare a set of spin-1 particles to be in the $|1\rangle_x$ state and then measure the z component of spin for these particles.
- (b) Using the simulation, prepare a set of particles to be in the $|1\rangle_x$ state and measure the x , y , and z components of spin of these particles. Draw probability histograms of the results for each spin-component-direction S_x , S_y , and S_z .