1 Measurement Probabilities (Brief)

A beam of spin-$\frac{1}{2}$ particles is prepared in the initial state

$$|\psi\rangle = \sqrt{\frac{2}{5}}\,|+\rangle_x - \sqrt{\frac{3}{5}}\,|-\rangle_x$$

(Note: this state is written in the S_x basis!)

(a) What are the possible measurement values if you measure the spin component S_x, and with what probabilities would they occur?

(b) What are the possible measurement values if you measure the spin component S_z, and with what probabilities would they occur?

2 Completeness Relation Change of Basis

(a) Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos\phi\,|\hat{x}\rangle + \sin\phi\,|\hat{y}\rangle \\ \left|\hat{\phi}\right\rangle &= -\sin\phi\,|\hat{x}\rangle + \cos\phi\,|\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle\hat{x}|\hat{s}\rangle\,, \quad \langle\hat{y}|\hat{s}\rangle\,, \quad \left\langle\hat{x}\middle|\hat{\phi}\right\rangle, \quad \left\langle\hat{y}\middle|\hat{\phi}\right\rangle$$

(b) Given a vector written in the polar basis

$$|\vec{v}\rangle = a\,|\hat{s}\rangle + b\left|\hat{\phi}\right\rangle$$

where a and b are known.

Express $|\vec{v}\rangle$ in the Cartesian basis,

$$|\vec{v}\rangle = c\,|\hat{x}\rangle + d\,|\hat{y}\rangle$$

by finding c and d

Hint: Use the completeness relation: $|\hat{x}\rangle\langle\hat{x}| + |\hat{y}\rangle\langle\hat{y}| = 1$

(c) Given a quantum state written in the S_z basis,

$$|\Psi\rangle = g\,|+\rangle + h\,|-\rangle\,,$$

express $|\Psi\rangle$ in the S_y basis. That is, find coefficients j and k such that

$$|\Psi\rangle = j\,|+\rangle_y + k\,|-\rangle_y\,.$$

3 Spin One Intro

The OSP Spins Laboratory simulation can also be used to explore spin-1 systems. The components of spin for these systems can be measured to be:

$\hbar$ (corresponding to the "+" port)
$0\hbar$ (corresponding to the "0" port)
$-\hbar$ (corresponding to the "-" port)

To switch the simulation to a spin-1 system, find the hyperlink about halfway down the page that says "Click here to switch".

(a) Draw and label a diagram of an experimental setup that would allow you to prepare a set of spin-1 particles to be in the $|1\rangle_x$ state and than measure the z component of spin for these particles.

(b) Using the simulation, prepare a set of particles to be in the $|1\rangle_x$ state and measure the x, y, and z components of spin of these particles. Draw probability histograms of the results for each spin-component-direction S_x, S_y, and S_z.