1 Histogram

Note: The states $|+\rangle_z$ and $|-\rangle_z$ are often written without the subscript z as $|+\rangle$ and $|-\rangle$. The subscripts are not omitted for the states $|+\rangle_x$, $|-\rangle_x$, $|+\rangle_y$, and $|-\rangle_y$.

A beam of spin-$\frac{1}{2}$ particles is prepared in the state:

$$|\psi\rangle = \frac{2}{\sqrt{13}}|+\rangle + i\frac{3}{\sqrt{13}}|-\rangle$$

(a) What are the possible measurement values if you measure the spin component S_z, and with what probabilities would they occur? *Check Beasts:* Check that you have the right "beast."

(b) What are the possible measurement values if you measure the spin component S_x, and with what probabilities would they occur? *Check Beasts:* Check that you have the right "beast."

(c) *Use Another Representation:* Plot histograms of the predicted measurement results from parts (a) and (b). Your histogram should be a vertical bar chart with the value of the spin component on the horizontal axis and the probability on the vertical axis.

2 Phase 2

Consider the three quantum states:

$$|\psi_1\rangle = \frac{4}{5}|+\rangle + i\frac{3}{5}|-\rangle$$
$$|\psi_2\rangle = \frac{4}{5}|+\rangle - i\frac{3}{5}|-\rangle$$
$$|\psi_3\rangle = -\frac{4}{5}|+\rangle + i\frac{3}{5}|-\rangle$$

(a) For each quantum state $|\psi_i\rangle$ given above, calculate the probabilities of obtaining $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ when measuring the spin component along the x-, y-, and z-axes (i.e.,S_x, S_x, and S_z).

(b) *Look For a Pattern (and Generalize):* Use your results from (a) to comment on the importance of the overall phase and of the relative phases of the quantum state vector.

3 Phase in Quantum States

In quantum mechanics, it turns out that the overall phase for a state does not have any physical significance. Therefore, you will need to become quick at rearranging the phase of various states. For each of the vectors listed below, rewrite the vector as an overall complex phase times a new vector whose first component is real and positive.

$$|D\rangle \doteq \begin{pmatrix} 7e^{i\frac{\pi}{6}} \\ 3e^{i\frac{\pi}{2}} \\ -1 \end{pmatrix} \quad |E\rangle \doteq \begin{pmatrix} i \\ 4 \end{pmatrix} \quad |F\rangle \doteq \begin{pmatrix} 2+2i \\ 3-4i \end{pmatrix}$$