1 Frequency

Consider a two-state quantum system with a Hamiltonian

$$\hat{H} \doteq \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \tag{1}$$

Another physical observable M is described by the operator

$$\hat{M} \doteq \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} \tag{2}$$

where c is real and positive. Let the initial state of the system be $|\psi(0)\rangle = |m_1\rangle$, where $|m_1\rangle$ is the eigenstate corresponding to the larger of the two possible eigenvalues of $\hat{M}$. What is the expectation value of M as a function of time? What is the frequency of oscillation of the expectation value of M?

2 Magnet

Consider a spin-1/2 particle with a magnetic moment. At time $t = 0$, the state of the particle is $|\psi(t = 0)\rangle = |+\rangle$.

(a) If the observable S_x is measured at time $t = 0$, what are the possible results and the probabilities of those results?

(b) Instead of performing the above measurement, the system is allowed to evolve in a uniform magnetic field $\vec{B} = B_0\,\hat{y}$. The Hamiltonian for a system in a uniform magnetic field $\vec{B} = B_0\,\hat{y}$ is $H = \omega_0\,S_y$. (You can treat ω_0 as a given parameter in your answers to the following two questions.)

- Calculate the state of the system after a time t and represent this state using the S_z basis.
- At time t, the observable S_x is measured, what is the probability that a value $\hbar/2$ will be found?

3 Wavefunction Calculations

Consider quantum particles in different 1-D potentials U(x).

For each of the following wavefunctions:

(i) Confirm that the wavefunction is normalizable by showing that the wavefunction goes to zero at $\pm\infty$.

(ii) Find the value of the overall constant that normalizes the state over all space.

(iii) Calculate the probability of locating the particle in the region $x = 0$ m to $x = 1$ m.

(iv) (*For the first two states* $|\psi_1\rangle$ *and* $|\psi_2\rangle$ *only*) Calculate the probability that the particle will be in a state

$$|\psi_{out}\rangle \doteq \begin{cases} 0 & x < 0 \\ \sqrt{2}\sin \pi x & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

after an unspecified measurement process.

(a) $|\psi_1\rangle \doteq \begin{cases} 0 & x < -1 \\ A & -1 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$

(b) $|\psi_2\rangle \doteq \begin{cases} 0 & x < 0 \\ Bx(x-3) & 0 \leq x \leq 3 \\ 0 & x > 3 \end{cases}$

(c) $|\psi_3\rangle \doteq \begin{cases} Ce^{k(x-1)} & x \leq 1 \\ Ce^{-k(x-1)} & x > 1 \end{cases}$