1 Completeness Relation Change of Basis

(a) Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos\phi\,|\hat{x}\rangle + \sin\phi\,|\hat{y}\rangle \\ \left|\hat{\phi}\right\rangle &= -\sin\phi\,|\hat{x}\rangle + \cos\phi\,|\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle\hat{x}|\hat{s}\rangle\,,\quad \langle\hat{y}|\hat{s}\rangle\,,\quad \left\langle\hat{x}\middle|\hat{\phi}\right\rangle,\quad \left\langle\hat{y}\middle|\hat{\phi}\right\rangle$$

(b) Given a vector written in the polar basis

$$|\vec{v}\rangle = a\,|\hat{s}\rangle + b\left|\hat{\phi}\right\rangle$$

where a and b are known. Find coefficients c and d such that

$$|\vec{v}\rangle = c\,|\hat{x}\rangle + d\,|\hat{y}\rangle$$

Do this by using the completeness relation:

$$|\hat{x}\rangle\langle\hat{x}| + |\hat{y}\rangle\langle\hat{y}| = 1$$

(c) Using a completeness relation, change the basis of the spin-1/2 state

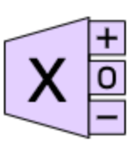
$$|\Psi\rangle = g\,|+\rangle + h\,|-\rangle$$

into the S_y basis. In otherwords, find j and k such that

$$|\Psi\rangle = j\,|+\rangle_y + k\,|-\rangle_y$$

2 Spin One Intro

The OSP Spins Laboratory simulation can also be used to explore spin-1 systems. The components of spin for these systems can be measured to be:
$\hbar$ (corresponding to the "+" port)
$0\hbar$ (corresponding to the "0" port)
$-\hbar$ (corresponding to the "-" port)



To switch the simulation to a spin-1 system, find the hyperlink about halfway down the page that says "Click here to switch".

(a) Draw and label a diagram of an experimental setup that would allow you to prepare a set of spin-1 particles to be in the $|1\rangle_x$ state and than measure the z component of spin for these particles.

(b) Using the simulation, prepare a set of particles to be in the $|1\rangle_x$ state and measure the x, y, and z components of spin of these particles. Draw probability histograms of the results for each spin-component-direction S_x, S_y, and S_z.

3 General State

Use a New Representation: Consider a quantum system with an observable A that has three possible measurement results: a_1, a_2, and a_3. States $|a_1\rangle$, $|a_2\rangle$, and $|a_3\rangle$ are eigenstates of the operator $\hat{A}$ corresponding to these possible measurement results.

(a) Using matrix notation, express the states $|a_1\rangle$, $|a_2\rangle$, and $|a_3\rangle$ in the basis formed by these three eigenstates themselves.

(b) The system is prepared in the state:

$$|\psi_b\rangle = N\left(1\,|a_1\rangle - 2\,|a_2\rangle + 5\,|a_3\rangle\right)$$

  (a) Staying in bra-ket notation, find the normalization constant.

  (b) Calculate the probabilities of all possible measurement results of the observable A. *Check "beasts."*

  (c) *Use a New Representation:* Plot a histogram of the predicted measurement results.

(c) In a different experiment, the system is prepared in the state:

$$|\psi_c\rangle = N\left(2\,|a_1\rangle + 3i\,|a_2\rangle\right)$$

  (a) Write this state in matrix notation and find the normalization constant.

  (b) Calculate the probabilities of all possible measurement results of the observable A. *Check "beasts".*

  (c) *Use a New Representation:* Plot a histogram of the predicted measurement results.

4 Spin One Interferometer Brief

Consider a spin 1 interferometer which prepares the state as $|1\rangle$, then sends this state through an S_x apparatus and then an S_z apparatus. For the four possible cases where a pair of beams or all three beams from the S_x Stern-Gernach analyzer are used, calculate the probabilities that a particle entering the last Stern-Gerlach device will be measured to have each possible value of S_z. Compare your theoretical calculations to results of the simulation. Make sure that you explicitly discuss your choice of projection operators.

Note: You do not need to do the first case, as we have done it in class.