1 Spin-1/2 Time Dependence Practice

Two electrons are placed in a magnetic field in the z-direction. The initial state of the first electron is $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$ and the initial state of the second electron is $\frac{1}{2}\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$.

(a) Find the probabilty of measuring each particle to have spin-up in the x-, y-, and z-directions at $t = 0$.

(b) Find the probabilty of measuring each particle to have spin-up in the x-, y-, and z-directions at some later time t.

(c) Calculate the expectation values for S_x, S_y, and S_z for each particle as functions of time.

(d) Are there any times when all the probabilities you have calculated are the same as they were at $t = 0$?

2 Frequency

Consider a two-state quantum system with a Hamiltonian

$$\hat{H} \doteq \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \tag{1}$$

Another physical observable M is described by the operator

$$\hat{M} \doteq \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} \tag{2}$$

where c is real and positive. Let the initial state of the system be $|\psi(0)\rangle = |m_1\rangle$, where $|m_1\rangle$ is the eigenstate corresponding to the larger of the two possible eigenvalues of $\hat{M}$. What is the expectation value of M as a function of time? What is the frequency of oscillation of the expectation value of M?

3 Magnet

Consider a spin-1/2 particle with a magnetic moment. At time $t = 0$, the state of the particle is $|\psi(t = 0)\rangle = |+\rangle$.

(a) If the observable S_x is measured at time $t = 0$, what are the possible results and the probabilities of those results?

(b) Instead of performing the above measurement, the system is allowed to evolve in a uniform magnetic field $\vec{B} = B_0\,\hat{y}$. The Hamiltonian for a system in a uniform magnetic field $\vec{B} = B_0\,\hat{y}$ is $H = \omega_0\,S_y$. (You can treat ω_0 as a given parameter in your answers to the following two questions.)

- Calculate the state of the system after a time t and represent this state using the S_z basis.
- At time t, the observable S_x is measured, what is the probability that a value $\hbar/2$ will be found?

4 Spin Three Halves Time Dependence

A spin-3/2 particle initially is in the state $|\psi(0)\rangle = |\frac{1}{2}\rangle$. This particle is placed in an external magnetic field so that the Hamiltonian is proportional to the $\hat{S}_x$ operator, $\hat{H} = \alpha\hat{S}_x \doteq \frac{\alpha\hbar}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$

(a) Find the energy eigenvalues and energy eigenstates for the system.

(b) Find $|\psi(t)\rangle$.

(c) List the outcomes of all possible measurements of S_x and find their probabilities. Explicitly identify any probabilities that depend on time.

(d) List the outcomes of all possible measurements of S_z and find their probabilities. Explicitly identify any probabilities that depend on time.