1 Matrix Elements and Completeness Relations

Writing an operator in matrix notation in its own basis is easy: it is diagonal with the eigenvalues on the diagonal.

What if I want to calculate the matrix elements using a different basis??

The eigenvalue equation tells me what happens when an operator acts on its own eigenstate. For example: $\hat{S}_y \left|\pm\right\rangle_y = \pm\frac{\hbar}{2}\left|\pm\right\rangle_y$

In Dirac bra-ket notation, to know what an operator does to a ket, I need to write the ket in the basis that is the eigenstates of the operator (in order to use the eigenvalue equation.)

One way to do this is to stick completeness relationships into the braket:

$$\left\langle +\right| \hat{S}_y \left|+\right\rangle = \left\langle +\right| (I)\hat{S}_y(I) \left|+\right\rangle$$

where I is the identity operator: $I = \left|+\right\rangle_{yy}\left\langle +\right| + \left|-\right\rangle_{yy}\left\langle -\right|$. This effectively rewrites the $\left|+\right\rangle$ in the $\left|\pm\right\rangle_y$ basis.

Find the top row matrix elements of the operator $\hat{S}_y$ in the S_z basis by inserting completeness relations into the brakets. (The answer is already on the Spins Reference Sheet, but I want you to demonstrate the calculation.)

2 Probabilities of Energy

(adapted from McIntyre Problem # 3.2)

(a) Show that the probability of a measurement of the energy is time independent for a general state:

$$\left|\psi(t)\right\rangle = \sum_n c_n(t)\left|E_n\right\rangle$$

that evolves due to a time-independent Hamiltonian.

(b) Show that the probabilities of measurements of other observables that commute with the Hamiltonian are also time independent (neither operator has degeneracy).

3 Spin Three Halves Operators

If a beam of spin-3/2 particles is input to a Stern-Gerlach analyzer, there are four output beams whose deflections are consistent with magnetic moments arising from spin angular momentum components of $\frac{3}{2}\hbar$, $\frac{1}{2}\hbar$, $-\frac{1}{2}\hbar$, and $-\frac{3}{2}\hbar$. For a spin-3/2 system:

(a) Write down the eigenvalue equations for the S_z operator.

(b) Write down the matrix representation of the S_z eigenstates in the S_z basis.

(c) Write down the matrix representation of the S_z operator in the S_z basis.

(d) Write down the eigenvalue equations for the S^2 operator. (The eigenvalues of the S^2 are $\hbar^2 s(s+1)$, where s is the spin quantum number. $S^2 = (S_x)^2 + (S_y)^2 + (S_z)^2$, which is proportional to the identify operator. For spin-3/2 system, $s = \frac{3}{2}$)

(e) Write down the matrix representation of the S^2 operator in the S_z basis. *Check Beasts:* Is your operator proportional to the identity operator?