

1 Matrix Refresher

Calculate the following quantities for the matrices:

$$A \doteq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad B \doteq \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \quad C \doteq \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

and the vector:

$$|D\rangle \doteq \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix}$$

- (a) AB
- (b) $\text{tr}(B)$
- (c) $A|D\rangle$
- (d) $\det(\lambda \mathcal{I} - A)$ where λ is a scalar.
- (e) C^{-1} (Hint: Geometrically, what is the C transformation? What transformation undoes what C does?)

2 Pauli Practice

The Pauli spin matrices σ_x , σ_y , and σ_z are defined by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

- (a) Show that each of the Pauli matrices is hermitian. (A matrix is hermitian if it is equal to its hermitian adjoint.)
- (b) Show that the determinant of each of the Pauli matrices is -1 .
- (c) Show that $\sigma_i^2 = \mathcal{I}$ for each of the Pauli matrices, i.e. for $i \in \{x, y, z\}$.

3 Hermitian Adjoins

Calculate the following quantities for the matrices:

$$A \doteq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad B \doteq \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \quad C \doteq \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

and the vectors:

$$|D\rangle \doteq \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix} \quad |E\rangle \doteq \begin{pmatrix} 1 \\ i \end{pmatrix} \quad |F\rangle \doteq \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- (a) $A^\dagger$
- (b) $|E\rangle^\dagger \equiv \langle E|$
- (c) $\langle D|A|D\rangle$
- (d) $(A|D\rangle)^\dagger$
- (e) Using explicit matrix multiplication (without using a theorem) verify that $(A|D\rangle)^\dagger = \langle D|A^\dagger$