

	Representation		
	Rectangular	Polar	Exponential
$Z$	$x + iy$	$r \cos \varphi + ir \sin \varphi$	$re^{i\varphi}$
$Z^*$			
$zz^* =  z ^2$			
$Z^2$			

## 1 Representations of Complex Numbers—Table

(1 pt each, 9 pts total) Fill out the table above that asks you to do several simple complex number calculations in rectangular, polar, and exponential representations.

## 2 Graphs of the Complex Conjugate

(2 pts each, 8 pts total)

For each of the following complex numbers, determine the complex conjugate, square, and norm. Then, plot and clearly label each  $z$ ,  $z^*$ , and  $|z|$  on an Argand diagram.

(a)  $z_1 = 4i - 3$

(b)  $z_2 = 5e^{-i\pi/3}$

(c)  $z_3 = -8$

(d) In a few full sentences, explain the geometric meaning of the complex conjugate and norm.

## 3 Euler's Formula I

(2, 4 pts)

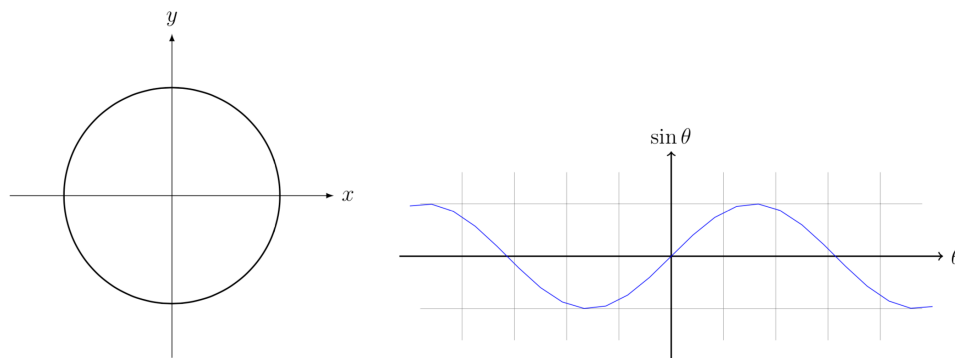
- (a) Use Euler's formula  $e^{i\phi} = \cos \phi + i \sin \phi$  and its complex conjugate to find formulas for  $\sin \phi$  and  $\cos \phi$ . In your physics career, you will often need to read these formula “backwards,” (i.e. notice one of these combinations of exponentials in a sea of other symbols and say, Ah ha! that is  $\cos \phi$ ). So, pay attention to the result of the homework problem!
- (b) Show that Euler's formula:

$$e^{i\phi} = \cos \phi + i \sin \phi$$

is true, by comparing the power series for the various terms.

## 4 Circle Trigonometry

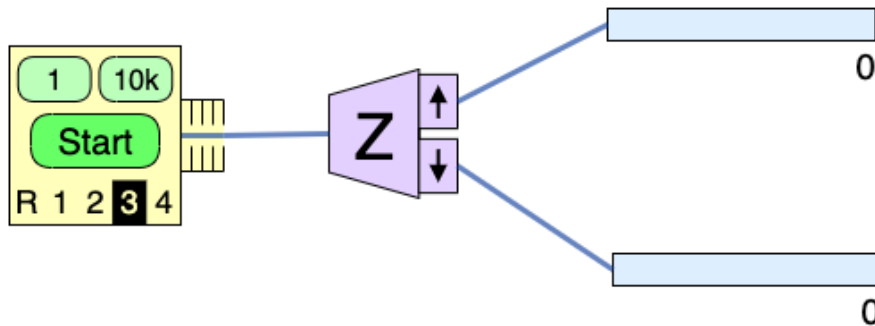
(4 pts) On the following diagrams, mark both  $\theta$  and  $\sin \theta$  for  $\theta_1 = \frac{5\pi}{6}$  and  $\theta_2 = \frac{7\pi}{6}$ . Write one to three sentences about how these two representations are related to each other. (For example, see: [this PHET](#))



## 5 Statistical Analysis of the Spins Sim

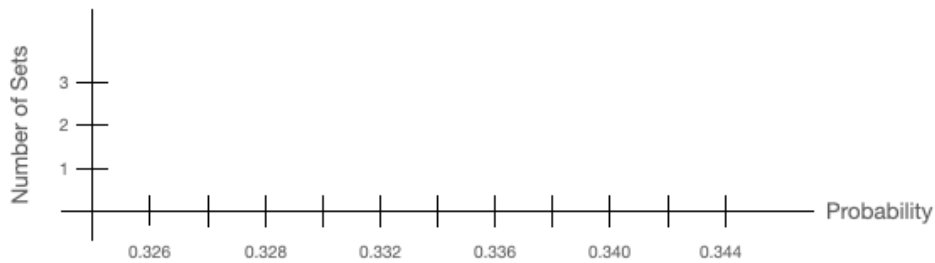
In the spins sim, the oven can be set to emit particles in a particular unknown prepared state (instead of in a random state).

- Set the oven to Unknown #3.
- Orient the analyzer in the  $z$ -direction.
- Perform 5 sets of 10,000 Stern-Gerlach experiments (10,000 particles are sent through a Stern-Gerlach Analyzer) and record the number of particles that end up in the top counter.
- For each set of experiments, calculate the probability that a single particle was measured to have  $S_z = +\hbar/2$ .



Do all of the following calculations by hand (you can use a calculator to help with the arithmetic).

- (a) Plot a histogram of the probabilities you measured for each set. Use a bin size of 0.002 for the horizontal axis. (Choose appropriate values on the horizontal axis. You don't need to plot the full possible values 0-1. You may use a computer to make the histogram or you can sketch it by hand.)



- (b) What is your best estimate of the probability that, when you measure  $S_z$  of a particle in the Unknown #3 state, you will get a result of  $+\hbar/2$ ? Mark this value on your histogram.