1 Spin Three Halves Operators

If a beam of spin-3/2 particles is input to a Stern-Gerlach analyzer, there are four output beams whose deflections are consistent with magnetic moments arising from spin angular momentum components of $\frac{3}{2}\hbar$, $\frac{1}{2}\hbar$, $-\frac{1}{2}\hbar$, and $-\frac{3}{2}\hbar$. For a spin-3/2 system:

(a) Write down the eigenvalue equations for the S_z operator.

(b) Write down the matrix representation of the S_z eigenstates in the S_z basis.

(c) Write down the matrix representation of the S_z operator in the S_z basis.

(d) Write down the eigenvalue equations for the S^2 operator. (The eigenvalues of the S^2 are $\hbar^2 s(s+1)$, where s is the spin quantum number. $S^2 = (S_x)^2 + (S_y)^2 + (S_z)^2$, which is proportional to the identify operator. For spin-3/2 system, $s = \frac{3}{2}$)

(e) Write down the matrix representation of the S^2 operator in the S_z basis. *Check Beasts:* Is your operator proportional to the identity operator?

2 Commute

Consider a three-dimensional state space. In the basis defined by three orthonormal kets $|1\rangle$, $|2\rangle$, and $|3\rangle$, the operators A and B are represented by:

$$A \doteq \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \quad B \doteq \begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}$$

where all the matrix elements are real.

(a) Do the operators A and B commute?

(b) Find the eigenvalues and normalized eigenvectors of both operators.

(c) Assume the system is initially in the state $|2\rangle$. Then the observable corresponding to the operator B is measured. What are the possible results of this measurement and the probabilities of each result? After this measurement, the observable corresponding to the operator A is measured. What are the possible results of this measurement and the probabilities of each result?

(d) *Interpret the Mathematical Model* How are questions (a) and (c) above related?

3 Matrix Elements and Completeness Relations

Writing an operator in matrix notation in its own basis is easy: it is diagonal with the eigenvalues on the diagonal.

What if I want to calculate the matrix elements using a different basis??

The eigenvalue equation tells me what happens when an operator acts on its own eigenstate. For example: $\hat{S}_y \left|\pm\right\rangle_y = \pm\frac{\hbar}{2}\left|\pm\right\rangle_y$

In Dirac bra-ket notation, to know what an operator does to a ket, I need to write the ket in the basis that is the eigenstates of the operator (in order to use the eigenvalue equation.)

One way to do this is to stick completeness relationships into the braket:

$$\left\langle+\right| \hat{S}_y \left|+\right\rangle = \left\langle+\right| (I)\hat{S}_y(I) \left|+\right\rangle$$

where I is the identity operator: $I = \left|+\right\rangle_{yy}\left\langle+\right| + \left|-\right\rangle_{yy}\left\langle-\right|$. This effectively rewrites the $\left|+\right\rangle$ in the $\left|\pm\right\rangle_y$ basis.

Find the top row matrix elements of the operator $\hat{S}_y$ in the S_z basis by inserting completeness relations into the brakets. (The answer is already on the Spins Reference Sheet, but I want you to demonstrate the calculation.)