1 Spin One Half Unknowns (Brief)

With the Spins simulation set for a spin 1/2 system, measure the probabilities of all the possible spin components for each of the unknown initial states $|\psi_3\rangle$ and $|\psi_4\rangle$. (Since $|\psi_3\rangle$ has already been covered in class, please only do $|\psi_4\rangle$)

(a) Use your measured probabilities to find each of the unknown states as a linear superposition of the S_z-basis states $|+\rangle$ and $|-\rangle$.

(b) *Articulate a Process:* Write a set of general instructions that would allow another student in next year's class to find an unknown state from measured probabilities.

(c) *Compare Theory with Experiment:* Design an experiment that will allow you to test whether your prediction for each of the unknown states is correct. Describe your experiment here, clearly but succinctly, as if you were writing it up for a paper. Do the experiment and discuss your results.

(d) *Make a Conceptual Connection:* In general, can you determine a quantum state with spin-component probability measurements in only two spin-component-directions (for example, z direction and y direction)?Why or why not?

2 Measurement Probabilities

A beam of spin-$\frac{1}{2}$ particles is prepared in the initial state

$$|\psi\rangle = \sqrt{\frac{2}{5}}\,|+\rangle_x - \sqrt{\frac{3}{5}}\,|-\rangle_x$$

(Note: this state is written in the S_x basis!)

(a) What are the possible results of a measurement of S_x, with what probabilities?

(b) Repeat part a for measurements of S_z.

(c) Suppose you start with a particle in the state given above, measure S_x, and happen to get $+\hbar/2$. You then take that same particle and measure S_z. What are the possible results and with what probability would you measure each possible result?

3 Phase 2

Consider the three quantum states:

$$|\psi_1\rangle = \frac{4}{5}\,|+\rangle + i\frac{3}{5}\,|-\rangle$$

$$|\psi_2\rangle = \frac{4}{5}\,|+\rangle - i\frac{3}{5}\,|-\rangle$$

$$|\psi_3\rangle = -\frac{4}{5}\,|+\rangle + i\frac{3}{5}\,|-\rangle$$

(a) For each quantum state $|\psi_i\rangle$ given above, calculate the probabilities of obtaining $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ when measuring the spin component along the x-, y-, and z-axes.

(b) *Look For a Pattern (and Generalize):* Use your results from (a) to comment on the importance of the overall phase and of the relative phases of the quantum state vector.

4 Pauli

(2 pts each)

The Pauli spin matrices σ_x, σ_y, and σ_z are defined by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

(a) Show that $\sigma_x\sigma_y = i\sigma_z$ and $\sigma_y\sigma_x = -i\sigma_z$. (Note: These identities also hold under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).

(b) The commutator of two matrices A and B is defined by $[A, B] \stackrel{\text{def}}{=} AB - BA$. Show that $[\sigma_x, \sigma_y] = 2i\sigma_z$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).

(c) The anti-commutator of two matrices A and B is defined by $\{A, B\} \stackrel{\text{def}}{=} AB + BA$. Show that $\{\sigma_x, \sigma_y\} = 0$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).