1 Matrix Refresher

Calculate the following quantities for the matrices:

$$A \doteq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \qquad B \doteq \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \qquad C \doteq \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

and the vector:

$$|D\rangle \doteq \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix}$$

(a) AB

(b) $\mathrm{tr}(B)$

(c) $A|D\rangle$

(d) $\det(\lambda\mathcal{I} - A)$ where λ is a scalar.

(e) C^{-1} (Hint: Geometrically, what is the C transformation? What transformation undoes what C does?)

2 Pauli Practice

The Pauli spin matrices σ_x, σ_y, and σ_z are defined by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

(a) Show that each of the Pauli matrices is hermitian. (A matrix is hermitian if it is equal to its hermitian adjoint.)

(b) Show that the determinant of each of the Pauli matrices is -1.

(c) Show that $\sigma_i^2 = \mathcal{I}$ for each of the Pauli matrices, i.e. for $i \in \{x, y, z\}$.

3 Hermitian Adjoints

Calculate the following quantities for the matrices:

$$A \doteq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \qquad B \doteq \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \qquad C \doteq \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

and the vectors:

$$|D\rangle \doteq \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix} \qquad |E\rangle \doteq \begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |F\rangle \doteq \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(a) $A^\dagger$

(b) $|E\rangle^\dagger \equiv \langle E|$

(c) $\langle D|A|D\rangle$

(d) $(A|D\rangle)^\dagger$

(e) Using explicit matrix multiplication (without using a theorem) verify that $(A|D\rangle)^\dagger = \langle D|A^\dagger$