1 Wavefunctions

Consider the following wave functions (over all space - not the infinite square well!):
$\psi_a(x) = Ae^{-x^2/3}$
$\psi_b(x) = B\frac{1}{x^2+2}$
$\psi_c(x) = C \text{ sech}\left(\frac{x}{5}\right)$ ("sech" is the hyperbolic secant function.)
In each case:

(a) normalize the wave function,

(b) find the probability that the particle is measured to be in the range $0 < x < 1$.

2 ISW Energy Measurement

A particle in an infinite square well potential has an initial state vector

$$|\Psi(0)\rangle = A\Big(\,|\phi_1\rangle - |\phi_2\rangle + i\,|\phi_3\rangle\,\Big)$$

where $|\phi_1\rangle, |\phi_2\rangle$, and $|\phi_3\rangle$ are the first three energy eigenstates.

(a) Determine A.

(b) At time $t = 0$, what are the possible outcomes of a measurement of energy, and with what probability would each possible outcome occur?

(c) What is the average value of energy one would measure at $t = 0$? In other words, what is the expectation value of energy at $t = 0$?

(d) What is the quantum state of this particle at some later time t?

(e) At time $t = \hbar/E_1$, what are the possible energies you would measure and with what probabilities would you measure them? *Check Beasts:* Verify that $\hbar/E_1$ is a time.

3 ISW Right Quarter

For a particle in an infinite square well from 0 to L, calculate the probability of finding the particle in the range $\dfrac{3L}{4} < x < L$ for each of the first three energy eigenstates.

4 ISW Expectation

Consider an infinite square well potential between 0 and L.

(a) Write down an expression for the nth energy eigenstate.

(b) Find the expectation value of position for the nth energy eigenstate.

(c) Find the uncertainty of position for the nth energy eigenstate.

(d) Find the expectation value of momentum for the nth energy eigenstate.

(e) Find the uncertainty of momentum for the nth energy eigenstate.