1 Stern Gerlach Explain

(a) Use words and equations to explain the key features of the Stern-Gerlach experiment.

(b) *Contrast Classical/Quantum* Explain what you would predict based only on classical physics for the Stern-Gerlach experiment and describe the difference between the classical prediction and the actual experimental results.

2 Diagonalization

(2 pts each)

(a) Let

$$|\alpha\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \text{and} \qquad |\beta\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Show that $|\alpha\rangle$ and $|\beta\rangle$ are orthonormal. (If a pair of vectors is orthonormal, that suggests that they might make a good basis.)

(b) Consider the matrix

$$C \doteq \begin{pmatrix}3 & 1\\1 & 3\end{pmatrix}$$

Show that the vectors $|\alpha\rangle$ and $|\beta\rangle$ are eigenvectors of C and find the eigenvalues. (Note that showing something is an eigenvector of an operator is far easier than finding the eigenvectors if you don't know them!)

(c) A operator is always represented by a diagonal matrix if it is written in terms of the basis of its own eigenvectors. What does this mean? Find the matrix elements for a new matrix E that corresponds to C expanded in the basis of its eigenvectors, i.e. calculate $\langle\alpha|C|\alpha\rangle$, $\langle\alpha|C|\beta\rangle$, $\langle\beta|C|\alpha\rangle$ and $\langle\beta|C|\beta\rangle$ and arrange them into a sensible matrix E. Explain why you arranged the matrix elements in the order that you did.

(d) Find the determinants of C and E. How do these determinants compare to the eigenvalues of these matrices?

3 Spin Matrix

(2 pts each)

The Pauli spin matrices σ_x, σ_y, and σ_z are defined by:

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \qquad \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \qquad \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics.

(a) By drawing pictures, convince yourself that the arbitrary unit vector $\hat{n}$ can be written as:

$$\hat{n} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

where θ and ϕ are the parameters used to describe spherical coordinates.

(b) Find the entries of the matrix $\hat{n}\cdot\vec{\sigma}$ where the "matrix-valued-vector" $\vec{\sigma}$ is given in terms of the Pauli spin matrices by

$$\vec{\sigma} = \sigma_x\,\hat{x} + \sigma_y\,\hat{y} + \sigma_z\,\hat{z}$$

and $\hat{n}$ is given in part (a) above.