	Representation		
	Rectangular	**Polar**	**Exponential**
z	$x+iy$	$r\cos\varphi + ir\sin\varphi$	$re^{i\varphi}$
z^*			
$zz^* = \lvert z\rvert^2$			
z^2			

1 Representations of Complex Numbers–Table

(1 pt each) Fill out the table above that asks you to do several simple complex number calculations in rectangular, polar, and exponential representations.

2 Graphs of the Complex Conjugate

(2 pts each)

For each of the following complex numbers, determine the complex conjugate, square, and norm. Then, plot and clearly label each z, z^*, and $|z|$ on an Argand diagram.

(a) $z_1 = 4i - 3$

(b) $z_2 = 5e^{-i\pi/3}$

(c) $z_3 = -8$

(d) In a few full sentences, explain the geometric meaning of the complex conjugate and norm.

3 Euler's Formula I

(2, 4 pts)

(a) Use Euler's formula $e^{i\phi} = \cos\phi + i\sin\phi$ and its complex conjugate to find formulas for $\sin\phi$ and $\cos\phi$. In your physics career, you will often need to read these formula "backwards," (i.e. notice one of these combinations of exponentials in a sea of other symbols and say, Ah ha! that is $\cos\phi$). So, pay attention to the result of the homework problem!

(b) Show that Euler's formula:

$$e^{i\phi} = \cos\phi + i\sin\phi$$

is true, by comparing the power series for the various terms.

4 Pauli Practice

(2 pts each)

The Pauli spin matrices σ_x, σ_y, and σ_z are defined by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

(a) Show that each of the Pauli matrices is hermitian. (A matrix is hermitian if it is equal to its hermitian adjoint.)

(b) Show that the determinant of each of the Pauli matrices is -1.

(c) Show that $\sigma_i^2 = \mathcal{I}$ for each of the Pauli matrices, i.e. for $i \in \{x, y, z\}$.

5 Pauli

(2 pts each)

The Pauli spin matrices σ_x, σ_y, and σ_z are defined by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

(a) Show that $\sigma_x\sigma_y = i\sigma_z$ and $\sigma_y\sigma_x = -i\sigma_z$. (Note: These identities also hold under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).

(b) The commutator of two matrices A and B is defined by $[A, B] \stackrel{\text{def}}{=} AB - BA$. Show that $[\sigma_x, \sigma_y] = 2i\sigma_z$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).

(c) The anti-commutator of two matrices A and B is defined by $\{A, B\} \stackrel{\text{def}}{=} AB + BA$. Show that $\{\sigma_x, \sigma_y\} = 0$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y$, $y \to z$, and $z \to x$).