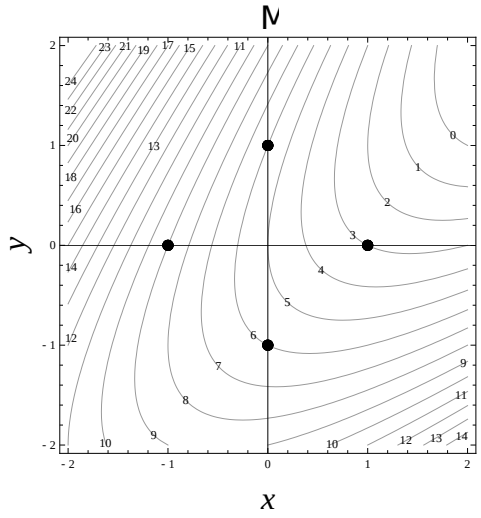

1 Contours

The figure shows a contour plot of a scalar field, $\mu(x, y)$. Assume that x and y are measured in meters and that μ is measured in kilograms. Four points are indicated on the plot.

(a) Determine $\frac{\partial \mu}{\partial x}$ and $\frac{\partial \mu}{\partial y}$ at each of the four points.

(b) On a printout of the figure, draw a qualitatively accurate vector at each point corresponding to the gradient of $\mu(x, y)$ using your answers to part a above. (How did you choose a scale for your vectors?)

Describe how the direction of the gradient vector is related to the contours on the plot and what property of the contour map is related to the magnitude of the gradient vector.

(c) Evaluate the gradient of $h(x, y) = (x + 1)^2 \left(\frac{x}{2} - \frac{y}{3}\right)^3$ at the point $(x, y) = (3, -2)$.

2 Gradient Practice

Find the gradient of each of the following functions:

(a)
$$f(x, y, z) = e^{(x+y)} + x^2 y^3 \ln \frac{x}{z} \tag{1}$$

(b)
$$\sigma(\theta, \phi) = \cos\theta \sin^2 \phi \tag{2}$$

(c)
$$\rho(s, \phi, z) = (s + 3z)^2 \cos\phi \tag{3}$$

3 Gauss's Law on a Cylindrical Shell

(6, 2, 2 pts)

Consider an infinite positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density $\rho(\vec{r}) = \alpha\, e^{(ks)^2}$.

Answer each of the following questions:

(a) Use Gauss's Law and symmetry arguments to find the electric field at each of the three radii below:

  (a) $s > b$

  (b) $a < s < b$

  (c) $s < a$

(b) What dimensions do α and k have?

(c) For $\alpha = 1$, $k = 1$, sketch the magnitude of the electric field as a function of s.

4 Flux Through a Cone

(6 pts)

Find the flux through a cone (with no cap!) of height H and radius R due to the vector field $\vec{F} = C\, z\, \hat{z}$. The cone has its vertex at the origin, opens upward symmetrically around the z-axis. Orient your area elements so that upward pointing fields contribute positive flux.