1 Constant Acceleration by Separation of Variables

(a) ***Calculate***: Treat Newton's 2nd law as a separable differential equation and solve for the velocity and position as a function of time of an object that is all of the following:

- moving in one dimension,
- not initially at the origin of coordinates,
- moving with a non-zero initial speed,
- experiences a constant force.

(b) ***Reflect***: Do your answers look familiar? If yes, from where? If not, how would you have to modify these equations to be similar to equations you know?

2 Separable ODE linear + constant

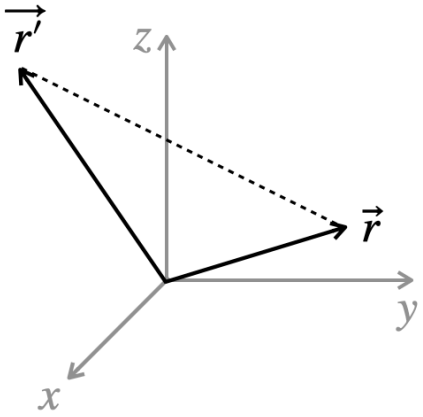
Solve the differential equation:

$\frac{dv}{dt} = -b - cv$ where $v(t = 0) = v_0$

3 Distance Formula in Curvilinear Coordinates

The distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ in rectangular coordinates.



(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{s^2 + s'^2 - 2ss'\cos(\phi - \phi') + (z - z')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{r'^2 + r^2 - 2rr'\left[\sin\theta \sin\theta' \cos(\phi - \phi') + \cos\theta\cos\theta'\right]}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(d) Now assume that $\vec{r}\,'$ and $\vec{r}$ are in the x-y plane. Simplify the previous two formulas.