1 Multiplicity of an ideal gas

(a) **(T3M.7)** The multiplicity of an ideal monatomic gas with N atoms, internal energy U, and volume V turns out to be roughly

$$\Omega(U, V, N) = CV^N U^{\left(\frac{3N}{2}\right)} \tag{1}$$

where C is a constant that depends on N alone. Use this expression, together with the fundamental definition of temperature, and they fundamental definition of entropy, to find U as a function of N and T for an ideal gas.

(b) **(From the GRE PhysicsSubject GR0177, given in 2001)**
Note 1: The irreversibility of this process tells you that entropy must go (up or down?).
Note 2: The gas constant, R, is equal to $N_{\text{Avogadro}} \cdot k_B$.

47. A sealed and thermally insulated container of total volume V is divided into two equal volumes by an impermeable wall. The left half of the container is initially occupied by n moles of an ideal gas at temperature T. Which of the following gives the change in entropy of the system when the wall is suddenly removed and the gas expands to fill the entire volume?

(A) $2nR \ln 2$

(B) $nR \ln 2$

(C) $\frac{1}{2}nR \ln 2$

(D) $-nR \ln 2$

(E) $-2nR \ln 2$

2 Helium heat capacity

In class, we assumed that all monatomic gases have 3 degrees of freedom ($f = 3$). In this question, we explore the possibility that a monatomic gas might have additional degrees of freedom due to the electrons orbiting the nucleus, or the rotation of the nucleus. To answer this question, you will need to use the equipartition theorem and understand how quantized energy levels affect the application of the equipartition theorem.

(a) An atom of helium can store energy by bumping its electron from its lowest orbital energy level to a higher orbital energy level. Moving an electron from the lowest state to the first excited state would store an energy of 24.6 eV (24.6 electron-volts). Give a quantitative explanation (i.e. by comparing quantities) that shows we can ignore this energy storage mode when calculating the heat capacity of helium gas **at ordinary temperatures.**

(b) The helium-4 nucleus can be modelled as a solid spherical object with mass m, radius r, and moment of inertia $I = (2/5)mr^2$. If the nucleus starts to rotate, it would have rotational kinetic energy $K_{\text{rotation}} = L^2/(2I)$, where L is the angular momentum. Usually the helium-4 nucleus has $L = 0$, however, it can be excited to a non-zero angular momentum state with $L \approx \hbar$, or $2\hbar$, or $3\hbar$, etc. (L is quantized). Give a quantitative explanation that shows we can ignore this energy storage mode when calculating the heat capacity of helium gas at ordinary temperatures.

3 Water and air heat capacity

In the last homework, you looked up the specific heat capacity of water and air ($c_{\text{p,water}} = 4.2$ J/(g.K) and $c_{\text{p,air}} = 1.0$ J/(g.K)) to analyze changes in the earth's climate. In this problem, you'll estimate $c_{p,\text{water}}$ and $c_{p,\text{air}}$ from first principles. Note: You'll use the equipartition theorem which is a coarse-grained alternative to using the full machinary of statistical mechanics, so, the answers might be off by a few %.

(a) For liquid water at room temperature, treat every oxygen atom and hydrogen atom as a point mass held in place by a 3-dimensional network of springs (like an Einstein solid). These "springs" arise from intra-molecular bonds (bonds within an H_2O molecule) and inter-molecular forces (the forces between neighboring H_2O molecules). Assume 1 gram of water and calculate the total number of degrees of freedom. Then find the total internal energy as a function of temperature, and the specific heat capacity at constant volume. Compare with measured value of $c_{\text{p,water}}$. Note: Liquid water doesn't expand/contract very much when heated, so $c_{\text{p,water}} \approx c_{\text{v,water}}$.

(b) The main components of air are nitrogen and oxygen. They are both diatomic gas molecules. You can model an O_2 gas molecule, or N_2 gas molecule, as two point masses connected by a stiff spring. The spring is so stiff that the energy quanta needed to excite the spring is bigger than $k_{\text{B}}T/2$ when $T = 300$ K. Therefore, you can treat the spring like a rigid rod. Calculate the total number of degrees of freedom in 1 gram of air. Then find the total internal energy as a function of temperature, and the specific heat capacity at constant volume. Compare with measured value of $c_{\text{v,air}} = 0.717$ J/(g.K). Note, $c_{\text{p,air}}$ is about 40% higher than $c_{\text{v,air}}$ because air held at constant pressure converts a sizable fraction of the heat into work when the gas expands.