

Rigid Rotor/Particle on a Sphere

	Ket Representation	Wave Function Representation	Matrix Representation
Hamiltonian	$\hat{H}$		$\begin{pmatrix} E_{0,0} & 0 & 0 & 0 & 0 & \dots \\ 0 & E_{1,1} & 0 & 0 & 0 & \dots \\ 0 & 0 & E_{1,0} & 0 & 0 & \dots \\ 0 & 0 & 0 & E_{1,-1} & 0 & \dots \\ 0 & 0 & 0 & 0 & E_{2,2} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 2\frac{\hbar^2}{2I} & 0 & 0 & 0 & \dots \\ 0 & 0 & 2\frac{\hbar^2}{2I} & 0 & 0 & \dots \\ 0 & 0 & 0 & 2\frac{\hbar^2}{2I} & 0 & \dots \\ 0 & 0 & 0 & 0 & 6\frac{\hbar^2}{2I} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of Hamiltonian			
Normalized Eigenstates of Hamiltonian		$Y_\ell^m(\theta, \phi) = (-1)^{(m+ m)/2} \sqrt{\frac{(2\ell+1)(\ell- m)!}{4\pi(\ell+ m)!}} P_\ell^m(\cos\theta) e^{im\phi}$	$ 0,0\rangle \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, 1,1\rangle \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, 1,0\rangle \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix}, 1,-1\rangle \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix}, \dots$
Coefficient of the energy eigenstate with quantum numbers ℓ, m	$c_{\ell,m} = \langle \ell, m \Psi \rangle$		
Probability of measuring $E_{\ell,m}$		$P\left(\frac{\hbar^2}{2I}\ell(\ell+1)\right) = \sum_{m=-\ell}^{\ell} \left \int_0^{2\pi} \int_0^\pi Y_\ell^m(\theta, \phi)^* \Psi(\theta, \phi) \sin\theta d\theta d\phi \right ^2$	$P(E_\ell) = \sum_{m=-\ell}^{\ell} \left \left(\dots \quad 1 \quad \dots \right) \begin{pmatrix} \vdots \\ c_{\ell,m} \\ \vdots \end{pmatrix} \right ^2$

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Operator for square of the angular momentum		$L^2 = -\hbar^2 \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{d^2}{d\phi^2} \right)$ $= -\hbar^2 \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\hbar^2 \sin^2 \theta} L_z^2 \right)$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 2\hbar^2 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 2\hbar^2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 2\hbar^2 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 6\hbar^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of L^2			
Normalized Eigenstates of L^2	$ \ell, m\rangle$		$ 0,0\rangle \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, 1,1\rangle \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, 1,0\rangle \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix}, 1,-1\rangle \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix}, \dots$
Coefficient of the eigenstates of L^2 with quantum numbers ℓ, m	$c_{\ell,m} = \langle \ell, m \Psi \rangle$	$c_{\ell,m} = \int_0^{2\pi} \int_0^\pi Y_\ell^m(\theta, \phi)^* \Psi(\theta, \phi) \sin \theta d\theta d\phi$	$(\cdots \quad 1 \quad \cdots) \begin{pmatrix} \vdots \\ c_{\ell,m} \\ \vdots \end{pmatrix}$
Probability of measuring $\hbar^2 \ell(\ell+1)$ for the square of the angular momentum			

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Operator for z-component of angular momentum	L_z		$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \hbar & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & -\hbar & 0 & \cdots \\ 0 & 0 & 0 & 0 & 2\hbar & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of L_z			
Normalized Eigenstates of L_z	$ \ell, m\rangle$	$Y_\ell^m(\theta, \phi) = (-1)^{(m+ m)/2} \sqrt{\frac{(2\ell+1)(\ell- m)!}{4\pi(\ell+ m)!}} P_\ell^m(\cos\theta) e^{im\phi}$	
Coefficient of m^{th} eigenstates of L_z	$c_{\ell,m} = \langle \ell, m \Psi \rangle$		$(\cdots \quad 1 \quad \cdots) \begin{pmatrix} \vdots \\ c_{\ell,m} \\ \vdots \end{pmatrix}$
Probability of measuring $m\hbar$ for z-component of angular momentum		$P(m\hbar) = \sum_{\ell= m }^{\infty} \left \int_0^{2\pi} \int_0^\pi Y_\ell^m(\theta, \phi)^* \Psi(\theta, \phi) \sin\theta d\theta d\phi \right ^2$	