**Operators & Functions**

For each of the following operators:

- Test each function to see if it is an eigenfunction of the operator.
- If it is, what is the eigenvalue?
- If it is not, can you write it as a superposition of functions that are eigenfunctions of that operator?

1. $\hat{p} = -i\hbar\frac{d}{dx}$

$$\psi_1(x) = Ae^{-ikx}$$
$$\psi_2(x) = Ae^{+ikx}$$
$$\psi_3(x) = A\sin(kx)$$

2. $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

$$\psi_1(x) = Ae^{-i\frac{p}{\hbar}x}$$
$$\psi_2(x) = Ae^{+i\frac{p}{\hbar}x}$$
$$\psi_3(x) = A\sin(\tfrac{p}{\hbar}x)$$

3. $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

$$\psi_1(x) = A\sin(kx)$$
$$\psi_2(x) = A\cos(kx)$$
$$\psi_3(x) = Ae^{ikx}$$

4. $\hat{S}_z \rightarrow \begin{pmatrix} \frac{\hbar}{2} & 0 \\ 0 & -\frac{\hbar}{2} \end{pmatrix}$

$$|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$|\psi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$|\psi_3\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Are superpositions of eigenfunctions of an operator themselves eigenfunctions of the same operator?

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