

1 Fourier Transform of a Gaussian

Consider the Gaussian wave function $f(x) = Ne^{-x^2/2\sigma^2}$.

- (a) Find the normalization constant N . Write a sentence describing the physical meaning of normalizing. (The identity $\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$ may prove helpful.)
- (b) Find the Fourier transform of $f(x)$ by hand. You will need to “complete the square”. Sense-making: Discuss how changing the constant σ changes the shape of both $f(x)$ and its Fourier transform.
- (c) Show that the Fourier transform of $f(x)$ is also normalized. (This is true for any function and is known as Parseval’s identity.) Write a sentence describing the physical meaning of normalizing in this case.

2 Fourier Transform of Cosine and Sine

- (a) Find the Fourier transforms of $f(x) = \cos kx$ and $g(x) = \sin kx$.
- (b) Find the Fourier transform of $g(x)$ using the formula for the Fourier transform of a derivative and your result for the Fourier transform of $f(x)$. Compare with your previous answer.
- (c) In quantum mechanics, the Fourier transform is the set of coefficients in the expansion of a quantum state in terms of plane waves, i.e. the function $\tilde{f}(k)$ is a continuous histogram of how much each functions e^{ikx} contributes to the quantum state. What does the Fourier transform of the function $\cos kx$ tell you about which plane waves make up this quantum state? Write a sentence or two about how this makes sense.

3 Fourier Transform of a Triangle

Consider a quantum mechanical wave packet shaped like a triangle:

$$f(x) = \begin{cases} \sqrt{\frac{3}{2\epsilon^3}}(x + \epsilon), & -\epsilon < x < 0 \\ -\sqrt{\frac{3}{2\epsilon^3}}(x - \epsilon), & 0 < x < \epsilon \\ 0, & \text{otherwise} \end{cases}$$

- (a) Show that the wave packet is normalized.
- (b) For three different values of ϵ , plot both the wave packet. (All three plots should be on the same axes.)
- (c) Find the Fourier Transform of the wave packet by hand. You may use technology of your choice to evaluate integrals, but do not use any built in Fourier Transform packages.

- (d) For three different values of ϵ , plot both the Fourier Transform of the wave packet. (All three plots should be on the same axes.)
- (e) As you change the value of ϵ so that the packet gets narrower and taller, what happens to the shape of the Fourier Transform?
- (f) Show that the Fourier transform is also (norm-squared) normalized.