

1 Cyclic Translation of 3 Distinguishable Beads

Consider 3 (distinguishable beads) arranged clockwise on a ring.

- Construct a matrix operator that translates (moves) each bead clockwise one “bead slot” along the ring.
- Write down an eigenvalue equation for this operator.
- Find eigenvalues and eigenvectors of this operator.

2 Adding Operators with a Common Set of Eigenvectors

- Consider linear operators $\hat{A}$ and $\hat{B}$ share a complete set of eigenvectors but with different eigenvalues.

$$\begin{aligned}\hat{A} |ab\rangle &= a |ab\rangle \\ \hat{B} |ab\rangle &= b |ab\rangle\end{aligned}$$

Now consider an operator $\hat{C}$ that is a linear combination of $\hat{A}$ and $\hat{B}$.
Show that $|ab\rangle$ is also an eigenvector of $\hat{C}$ and find its eigenvalues.

3 $\hat{S}^\uparrow$ and $\hat{S}^\downarrow$

Consider the operators:

$$\begin{aligned}\hat{S}^\uparrow &\doteq \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \\ \hat{S}^\downarrow &= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\end{aligned}$$

- Show that $\hat{S}^\uparrow$ and $\hat{S}^\downarrow$ are inverses of each other.
- Show that the eigenvectors of $\hat{S}^\downarrow$ are also eigenvectors of $\hat{S}^\uparrow$.
- Find the three eigenvalues of the operator $(\hat{S}^\uparrow + \hat{S}^\downarrow)$ and show that at least one of them checks out.