

1 Wavefunctions

(2*2, 2*2 points)

Consider the following wave functions, each describing a particle in one dimension and defined over all space (i.e., $-\infty < x < \infty$), not confined to an infinite square well.

$$\psi_a(x) = Ae^{-x^2/3}$$

$$\psi_b(x) = B\frac{1}{x^2+2}$$

For each wave function:

- Determine the normalization constant.
- If the particle's position is measured, what is the probability of finding it in the region $0 < x < 1$?

2 ISW Energy Measurement wave function form

(2*7 points)

A particle in an infinite square well potential has an initial state vector

$$|\Psi(t=0)\rangle = A(|\phi_1\rangle - |\phi_2\rangle + i|\phi_3\rangle)$$

where $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ are the first three energy eigenstates (i.e., $n = 1, 2, 3$).

- Determine A .
- Write the initial state $|\Psi(t=0)\rangle$ in wavefunction form.
- At time $t = 0$, if an energy measurement is performed, what are the possible energy values, and with what probability would each possible value be obtained?
- What is the expectation value of the energy of this particle at $t = 0$?
- What is the quantum state of this particle at some later time t ?
- At some later time t , write an expression (do not evaluate) for the probability that a position measurement yields a result in the first half of the well, $0 < x < \frac{L}{2}$.
- At time $t = \hbar/E_1$, if the energy of the particle is measured, what possible energy values may be obtained, and with what probability would each possible value be obtained? *Check Beasts:* Verify that $\hbar/E_1$ is a time.