



1 Q factor of a Resonance Circuit

We define a “quality factor”, $Q \equiv \frac{\omega_0}{2\beta}$, and use it as a measure of cycles in a free (undriven), damped oscillator before the oscillation decays to some smaller amplitude. The larger the number of cycles, the larger the Q . Now let’s see what this quantity translates to in a driven, damped oscillator. Use the example of charge amplitude $|q|$ for a series LRC circuit.

- Show that at both frequencies $\omega = \omega_0 + \beta$ and $\omega = \omega_0 - \beta$, the magnitude of the charge response $|q|$ is $\frac{|q|_{\text{max}}}{\sqrt{2}}$.
(Hint – You don’t need the definition of $|q|$ in terms of L, R, C).
- At these frequencies, how large is the energy in the capacitor compared to the maximal energy at $\omega \approx \omega_0$?
- Given the above, operationally, how would you measure the Q of a resonant circuit? What is Q for your circuit?

The graphs below may help with a visual feel for the quantities discussed above:

2 Steady State Solutions for the LRC Circuit

- Write the equation of motion governing the charge on the capacitor in a series LRC circuit driven by an external sinusoidal voltage. Identify all parameters in your equation.
- Find a steady-state solution (or particular solution) for the *current* in the circuit. After what time is the steady state the only relevant part of the solution, *i.e.*, after what time has the transient solution decayed for the circuit you are working with?
- Find the steady state solution for $\frac{dI}{dt}$ in this circuit.