1 Bottle in a Bottle Part 2

Consider the bottle-in-a-bottle problem in a previous problem set, summarized here. A small bottle of helium is placed inside a large bottle, which otherwise contains vacuum. The inner bottle contains a slow leak, so that the helium leaks into the outer bottle. The inner bottle contains one tenth the volume of the outer bottle. The outer bottle is insulated.

The volume of the small bottle is 0.001 m^3 and the volume of the big bottle is 0.01 m^3. The initial state of the gas in the small bottle was $p = 106$ Pa and its temperature $T = 300$ K. Approximate the helium gas as an ideal gas of equations of state $pV = Nk_BT$ and $U = \frac{3}{2}Nk_BT$.

(a) How many molecules of gas are initially in the small bottle? What is the final temperature of the gas after the pressures have equalized?

(b) Compute the change of entropy ΔS between the initial state (gas in the small bottle) and the final state (gas in both bottles, pressures equalized). Do not use the Sackur-Tetrode equation, use an alternative method.

(c) Discuss your results.

2 Melting ice lab questions

These questions relate to the in-class activity where you put ice (prepared at zero degree celcius) into some warm water. You insulated the ice and water inside nested polystyrene cups and closed the lid. You waited a few minutes for the system to reach equilibrium. At equilibruium, all the melted ice and water were at the same temperature. During class, you started to calculate a prediction for this final temperature.

(a) **Prediction** What is your prediction for the final temperature? List the measured quantities that you used in your calcultion. Show your work.

(b) **Measurement** What final temperature did you measure? Comment on the magnitude and sources of errors in your experiment and in your prediction.

(c) **Change in entropy** Calculate the total change in entropy that occured during the overall process. *Initial state*: warm water and ice. *Final state*: cool water at a uniform temperature.

3 Adiabatic Ideal Gas

Consider the adiabatic expansion of a simple ideal gas (adiabatic means that no energy is transfered by heating). The internal energy is given by

$$U = C_v T \tag{1}$$

where you may take C_v to be a constant—although for a polyatomic gas such as oxygen or nitrogen, it is temperature-dependent. The ideal gas law

$$pV = Nk_B T \tag{2}$$

determines the relationship between p, V and T. You may take the number of molecules N to be constant.

(a) Use the first law to relate the inexact differential for work to the exact differential dT for an adiabatic process.

(b) Find the total differential dT where T is a function $T(p, V)$.

(c) In the previous two sections, we found two formulas involving dT. Use the additional definition of work $đW = -pdV$ to solve for the relationship between p, dp, V and dV for an adiabatic process.

(d) Integrate the above differential equation to find a relationship between the initial and final pressure and volume for an adiabatic process.

4 Adiabatic Compression

A diesel engine requires no spark plug. Rather, the air in the cylinder is compressed so highly that the fuel ignites spontaneously when sprayed into the cylinder.

In this problem, you may treat air as an ideal gas, which satisfies the equation $pV = Nk_B T$. You may also use the property of an ideal gas that the internal energy depends only on the temperature T, i.e. the internal energy does not change for an isothermal process. For air at the relevant range of temperatures the heat capacity at fixed volume is given by $C_V = \frac{5}{2}Nk_B$, which means the internal energy is given by $U = \frac{5}{2}Nk_B T$.

Note: Looking up the formula in a textbook is *not* considered a solution at this level. Use only the equations given, fundamental laws of physics, and results you might have already derived from the same set of equations in other homework questions.

(a) If the air is initially at room temperature (taken as $20^o C$) and is then compressed adiabatically to $\frac{1}{15}$ of the original volume, what final temperature is attained (before fuel injection)?

(b) By what factor does the pressure increase (before fuel injection)?