1 Linear Quadrupole (w/ series)

parts b) and c) only

Consider a collection of three charges arranged in a line along the z-axis: charges $+Q$ at $z = \pm D$ and charge $-2Q$ at $z = 0$.

(a) Find the electrostatic potential at a point $\vec{r}$ in the xy-plane at a distance s from the center of the quadrupole. The formula for the electrostatic potential V at a point $\vec{r}$ due to a charge Q at the point $\vec{r}'$ is given by:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{|\vec{r} - \vec{r}'|}$$

(b) Assume $s \gg D$. Find the first two non-zero terms of a power series expansion to the electrostatic potential you found in the first part of this problem.

(c) A series of charges arranged in this way is called a linear quadrupole. Why?

2 Potential from a Finite Disk

In this problem, in all cases, you are expected to evaluate any integrals in your answers.

(a) Starting with the integral expression for the electrostatic potential due to a ring of charge, find the value of the potential everywhere along the axis of symmetry.

(b) Find the electrostatic potential everywhere along the axis of symmetry due to a finite disk of charge with uniform (surface) charge density σ.

3 Potential from a Cone

A conical surface (an empty ice-cream cone) carries a uniform charge density σ. The height of the cone is H, and the radius of the top is R.

(a) Find the potential at point $\vec{r}$ (in the center of the opening of the cone), letting the potential at infinity be zero.

(b) Evaluate your integral in the special case that $H = R$. You may use a computer algebra system.