1 Cone Surface

(a) Find dA on the surface of an (open) cone in both cylindrical and spherical coordinates. Hint: Be smart about how you coordinatize the cone.

(b) Using integration, find the surface area of an (open) cone with height H and radius R. Do this problem in both cylindrical and spherical coordinates.

2 Total Charge (HW)

For each case below, find the total charge.

(a) A positively charged (dielectric) spherical shell of inner radius a and outer radius b with a spherically symmetric internal charge density

$$\rho(\vec{r}) = 3\alpha\, e^{(kr)^3}$$

(b) A positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density

$$\rho(\vec{r}) = \alpha\, \frac{1}{s}\, e^{ks}$$

3 Mass of a Slab

Determine the total mass of each of the slabs below.

(a) A square slab of side length L with thickness h, resting on a table top at $z = 0$, whose mass density is given by

$$\rho = A\pi \sin\left[\tfrac{\pi z}{h}\right].$$

(b) A square slab of side length L with thickness h, resting on a table top at $z = 0$, whose mass density is given by

$$\rho = 2A\Big[\Theta(z) - \Theta(z-h)\Big]$$

(c) An infinitesimally thin square sheet of side length L, resting on a table top at $z = 0$, whose surface density is given by $\sigma = 2Ah$.

(d) An infinitesimally thin square sheet of side length L, resting on a table top at $z = 0$, whose mass density is given by $\rho = 2Ah\,\delta(z)$.

(e) What are the dimensions of A?

(f) Write several sentences comparing your answers to the different cases above.

4 Spherical Shell Step Functions

One way to write volume charge densities without using piecewise functions is to use step (Θ) or delta (δ) functions.

Consider a spherical shell with charge density

$$\rho(\vec{r}) = \alpha 3 e^{(kr)^3}$$

between the inner radius a and the outer radius b. The charge density is zero everywhere else.

(a) What are the dimensions of the constants α and k?

(b) By hand, sketch a graph the charge density as a function of r for $\alpha > 0$ and $k > 0$.

(c) Use step functions to write this charge density as a single function valid everywhere in space.