Part I
Using the applet at Inner Products of Functions:

1. Sketch the function $f(x)g(x)$ for the default functions.
2. Make a list of properties of the functions $f(x)$ and $g(x)$ that you used to make your sketch. Pay attention to special cases and symmetries.
3. Check your sketch by clicking on the green check box, i.e. refer to authority.
4. Plug other functions into the applet and check that your list of properties is accurate and complete, i.e. check many cases.

Part II
Use the applet to find the values of the following integrals for integer m and m':

$$\int_0^{2\pi} \sin mx \ \sin m'x \, dx$$

$$\int_0^{2\pi} \sin mx \ \cos m'x \, dx$$

$$\int_0^{2\pi} \cos mx \ \cos m'x \, dx$$

Part III
Do a simple change of variables in your integrals to convince yourself of the following:
For integer m and m'

$$\int_0^{L} \sin \tfrac{2\pi mx}{L} \ \sin \tfrac{2\pi m'x}{L} \, dx = \begin{cases} \frac{L}{2} \text{ if } m = m' \\ 0 \text{ if } m \neq m' \end{cases}$$

$$\int_0^{L} \sin \tfrac{2\pi mx}{L} \ \cos \tfrac{2\pi m'x}{L} \, dx = \begin{cases} 0 \text{ if } m = m' \\ 0 \text{ if } m \neq m' \end{cases}$$

$$\int_0^{L} \cos \tfrac{2\pi mx}{L} \ \cos \tfrac{2\pi m'x}{L} \, dx = \begin{cases} \frac{L}{2} \text{ if } m = m' \neq 0 \\ L \text{ if } m = m' = 0 \\ 0 \text{ if } m \neq m' \end{cases}$$

Hint: Recall that the function transformation $f(x) \to f(\alpha x)$ shrinks or expands the function $f(x)$ along the x-axis. See GMM: Function Transformations

Part IV

Compare your results for sines and cosines integrated over a whole period (this example) to what you know about the energy eigenstates of a quantum infinite square well, i.e. compare to a known example. How are these examples the same or different? Can you use the applet to explore the infinite square well case?