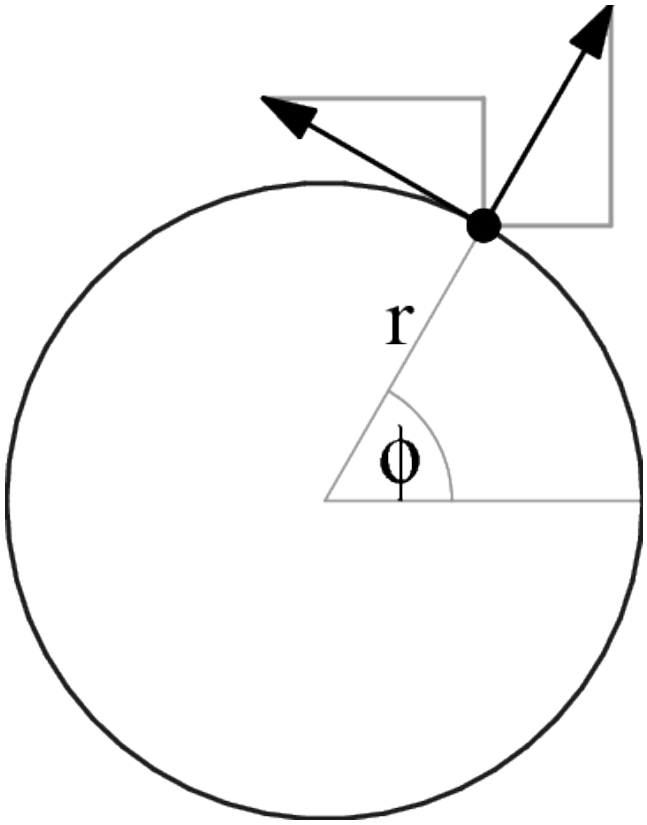


In rectangular coordinates, the natural unit vectors are $\{\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}\}$, which point in the direction of increasing x and y, respectively. Similarly, in polar coordinates the natural unit vectors are $\hat{\boldsymbol{r}}$, which points in the direction of increasing r, and $\hat{\boldsymbol{\phi}}$, which points in the direction of increasing ϕ.

The *unit tangent vector* to a parametric curve is the unit vector tangent to the curve which points in the direction of increasing parameter. The *principal unit normal vector* to a parametric curve is the unit vector perpendicular to the curve "in the direction of bending", which is the direction of the *derivative* of the unit tangent vector.

1. Consider the parametric curve $\vec{\boldsymbol{r}} = 3\cos\phi\,\hat{\boldsymbol{x}} + 3\sin\phi\,\hat{\boldsymbol{y}}$ with $\phi \in [0, 2\pi]$. Calculate the unit tangent vector $\hat{\boldsymbol{T}}$ and the principal unit normal vector $\hat{\boldsymbol{N}}$ for this curve in terms of $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$.

2. Consider a circle of radius 3 centered at the origin. Determine the unit tangent vector $\hat{\boldsymbol{T}}$ and the principal unit normal vector $\hat{\boldsymbol{N}}$ for this curve in terms of $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\phi}}$.

3. Compare your answers.