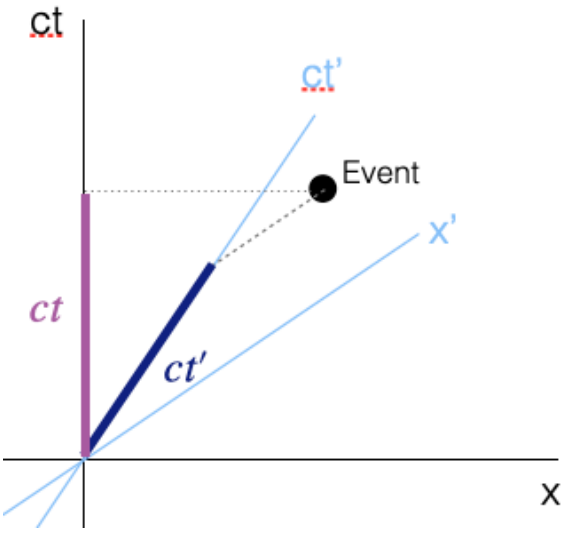
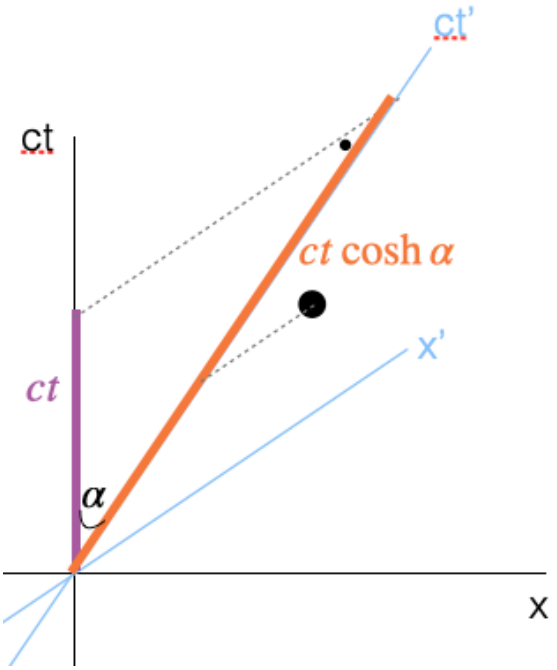
Here is a geometric derivation of the Lorentz Transformation using hyperbola geometry:

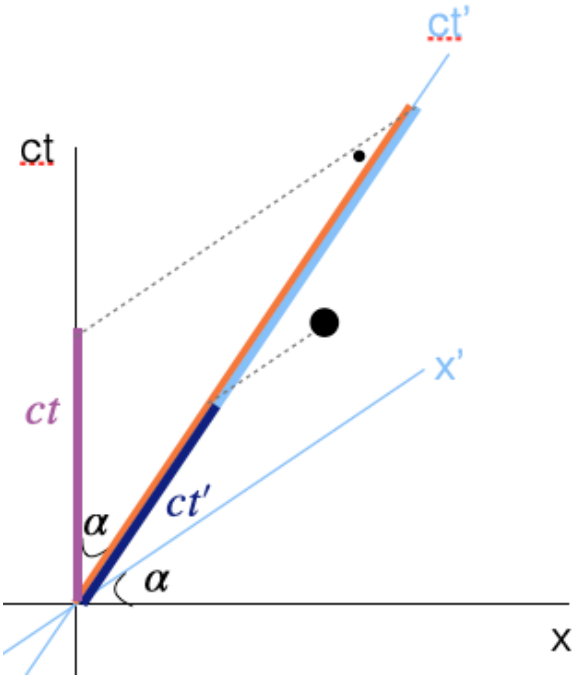
$$\begin{bmatrix} x' \\ ct' \end{bmatrix} = \begin{bmatrix} \cosh\alpha & -\sinh\alpha \\ -\sinh\alpha & \cosh\alpha \end{bmatrix} \begin{bmatrix} x \\ ct \end{bmatrix}$$

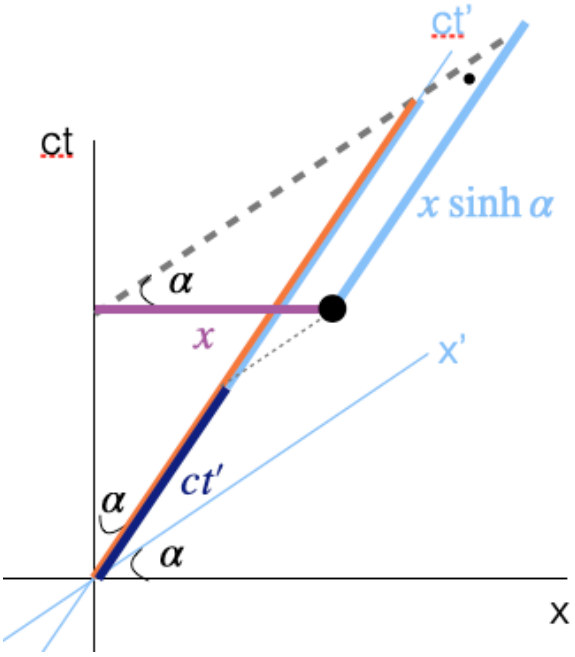
On the spacetime diagrams, the large black dot is the event we're trying to describe. The small black dot indicates a right angle for a hyperbolic triangle.

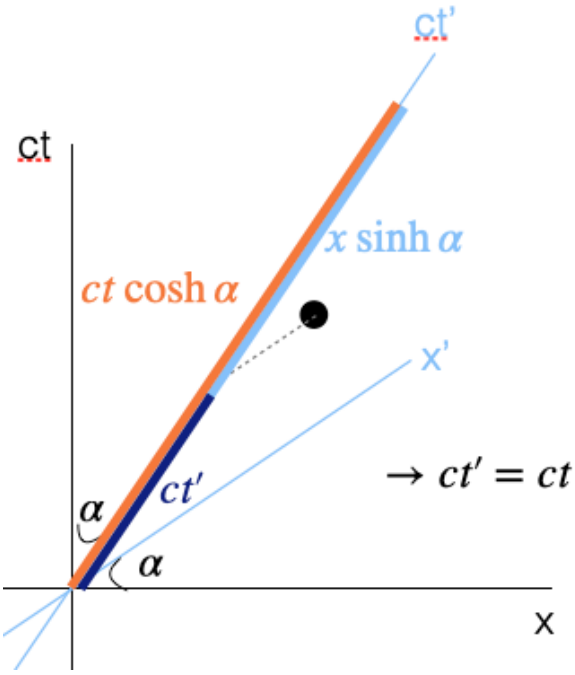
Starting with the time coordinate:



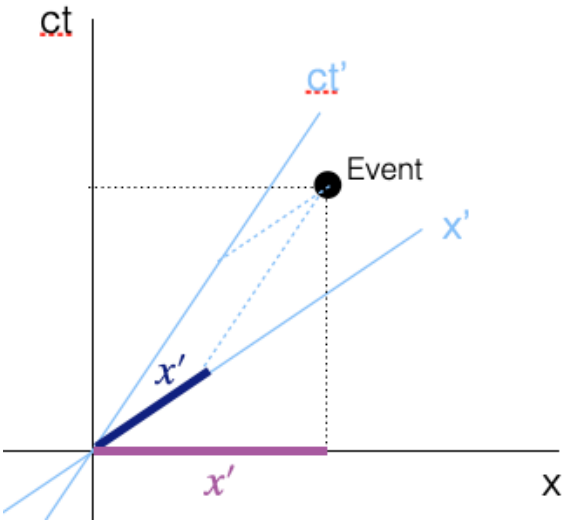

ct'
ct
x'
ct
ct'
α
α
x

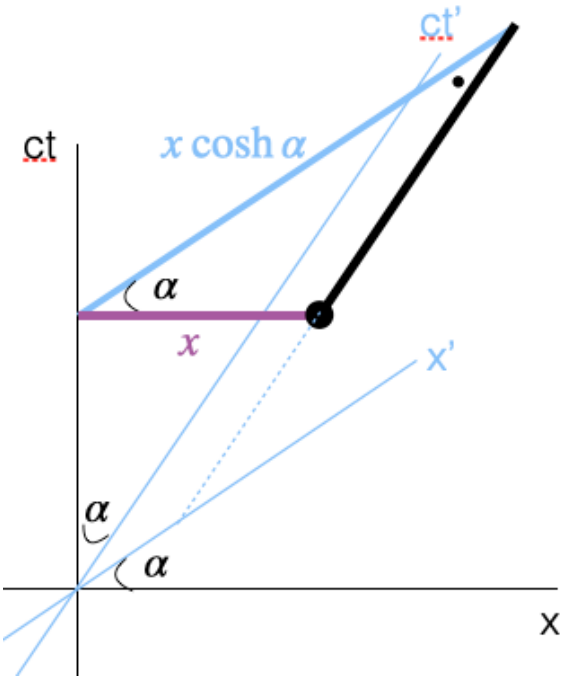

ct'
ct
x sinh α
α
x
x'
α
ct'
α
x

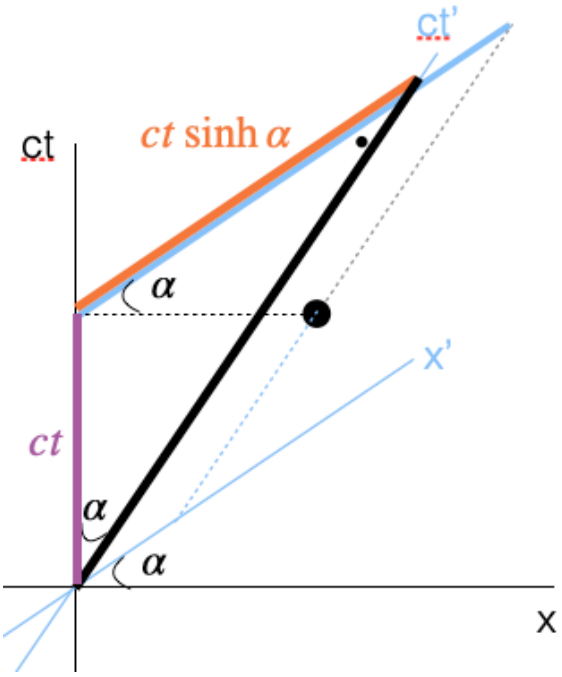


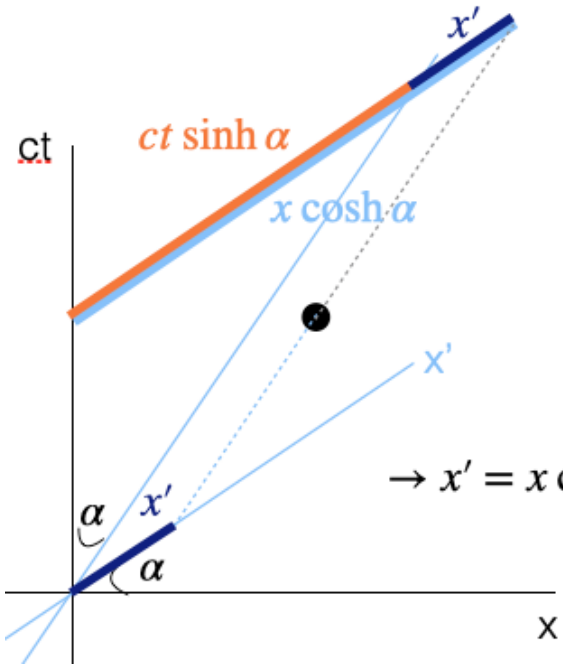

$$\rightarrow ct' = ct\cosh\alpha - x\sinh\alpha$$

Now thinking about the spatial coordinate:




ct'
ct
x cosh α
α
x
x'
α
α
x


ct'
ct
ct sinh α
α
x'
ct
α
α
x




$$\rightarrow x' = x\cosh\alpha - ct\sinh\alpha$$