Power Series Coefficients

Consider the power series:

$$f(z) = \sum_{n=0}^{\infty} c_n (z - z_0)^n$$

expanded around the point z_0. The coefficients are found from the formula:

$$c_n = \frac{f^{(n)}(z_0)}{n!}$$

1. Find the first four non-zero coefficients for $\sin\theta$ expanded around the origin.

2. Write out the series approximation for $\sin\theta$ expanded around the origin including only the first four non-zero terms.

   $\sin\theta \approx$

3. Plot $\sin\theta$ and your series approximation term by term and notice where the approximation is good.

4. Find the 4th order coefficients for $\sin\theta$ expanded around $\theta_0 = \pi/6$.

5. Write out the series approximation, correct to 4th order, for $\sin\theta$ expanded around $\theta_0 = \pi/6$.

   $\sin\theta \approx$

6. Plot $\sin\theta$ and your series approximation term by term and notice where the approximation is good.

7. What does it mean to write a series expansion around the point a?

8. Briefly describe in words how to expand the series approximation for a function, correct to 4th order.