Student handout

On the same set of axes, plot:

$$f(x) = x$$
$$g(x) = x^2$$
$$h(x) = f(x) + g(x)$$

1 Instructor's Guide

This SWBQ is really important. It is surprising how many students are unable to complete the prompt without an explanation. The idea of adding two functions pointwise is the geometry behind the superposition principle, power series, Fourier series, etc. Position this activity early in the upper-division curriculum and repeat as needed.

Note: Since many students only every plot the sum of two functions using technology, a few will never have noticed that this is what the technology is doing. If this is the case, they will miss the whole point of Fourier series.

1.1 The Prompt

On a single set of axes, sketch two functions. If this is part of the introduction to power series, a good choice is a straight line and a parabola.

If this SWBQ is part of an introduction to Fourier series, a good choice is two periodic functions with the same period.

At least one function should take on both positive and negative values. Ask the students to sketch these two functions on their small whiteboards and then to sketch the sum on the same set of axes.

1.2 Wrap-Up

Make sure that all students understand that the plus sign in the expression

$$f(x) + g(x)$$

means to add the functions "pointwise." You will probably need to discuss how to pick a value of x, locate the values of $f(x)$ and $g(x)$ on the graphs, add those two numbers and put that number on the sketch. Repeat.

This geometric definition will become the meaning of "addition" in applications like Fourier series, and more generically, is the definition of "addition" when they view a set of functions with a particular boundary condition as a vector space.

1.3 Extension

In Static Fields, we have followed this example with sketching a cross-section of the potential due to two point charges (as two separate functions on the same axes) and then asked the students to sketch the sum using the superposition principle. Combining the math and physics examples and languages may be particularly powerful.