1 Gibbs entropy is extensive

Consider two *noninteracting* systems A and B. We can either treat these systems as separate, or as a single combined system AB. We can enumerate all states of the combined by enumerating all states of each separate system. The probability of the combined state (i_A, j_B) is given by $P_{ij}^{AB} = P_i^A P_j^B$. In other words, the probabilities combine in the same way as two dice rolls would, or the probabilities of any other uncorrelated events.

(a) Show that the entropy of the combined system S_{AB} is the sum of entropies of the two separate systems considered individually, i.e. $S_{AB} = S_A + S_B$. This means that entropy is extensive. Use the Gibbs entropy for this computation. You need make no approximation in solving this problem.

(b) Show that if you have N identical non-interacting systems, their total entropy is NS_1 where S_1 is the entropy of a single system.

Note In real materials, we treat properties as being extensive even when there are interactions in the system. In this case, extensivity is a property of large systems, in which surface effects may be neglected.