Student handout

In this course, we will examine a mathematically tractable and physically useful problem - that of two bodies interacting with each other through a *central* force, i.e. a force that has two characteristics:
Definition of a Central Force:

1. A central force depends only on the separation distance between the two bodies,
2. A central force points along the line connecting the two bodies.

The most common examples of this type of force are those that have $\frac{1}{r^2}$ behavior, specifically the Newtonian gravitational force between two point (or spherically symmetric) masses and the Coulomb force between two point (or spherically symmetric) electric charges. Clearly both of these examples are idealizations - neither ideal point masses or charges nor perfectly spherically symmetric mass or charge distributions exist in nature, except perhaps for elementary particles such as electrons. However, deviations from ideal behavior are often small and can be neglected to within a reasonable approximation. (Power series to the rescue!) Also, notice the difference in length scale: the archetypal gravitational example is planetary motion - at astronomical length scales; the archetypal Coulomb example is the hydrogen atom - at atomic length scales.

The two solutions to the central force problem - classical behavior exemplified by the gravitational interaction and quantum behavior exemplified by the Coulomb interaction - are quite different from each other. By studying these two cases together in the same course, we will be able to explore the strong similarities and the important differences between classical and quantum physics.

Two of the unifying themes of this topic are the conservation laws:

- Conservation of Energy
- Conservation of Angular Momentum

The classical and quantum systems we will explore both have versions of these conservation laws, but they come up in the mathematical formalisms in different ways. You should have covered energy and angular momentum in your introductory physics course, at least in simple, classical mechanics cases. Now is a great time to review the definitions of energy and angular momentum, how they enter into dynamical equations (Newton's laws and kinetic energy, for example), and the conservation laws.

In the classical mechanics case, we will obtain the equations of motion in three equivalent ways,

- using Newton's second law,
- using Lagrangian mechanics,
- using energy conservation.

so that you will be able to compare and contrast the methods. The Newtonian approach is the most straightforward and naive, but it suggests changes of coordinates that inform the other methods. The Lagrangian and energy conservation approaches are slightly more sophisticated in that they exploit more of the symmetries from the beginning.

We will also consider forces that depend on the distance between the two bodies in ways other than $\frac{1}{r^2}$ and explore the kinds of motion they produce.