> **Student handout** A pretzel is to be dipped in chocolate. The pretzel is in the shape of a quarter circle, consisting of a straight segment from the origin to the point (2,0), a circular arc from there to (0,2), followed by a straight segment back to the origin; all distances are in centimeters. The (linear) density of chocolate on the pretzel is given by $\lambda = 3(x^2 + y^2)$ in grams per centimeter. Find the total amount of chocolate on the pretzel.

0.0.1 Main ideas

- Calculating (scalar) line integrals.
- Use what you know!

0.0.2 Prerequisites

- Familiarity with $d\vec{r}$.
- Familiarity with "Use what you know" strategy.

0.0.3 Warmup

It is *not* necessary to explicitly introduce scalar line integrals, before this lab; figuring out that the (scalar) line element must be $|d\vec{r}|$ can be made part of the activity (if time permits).

0.0.4 Props

- whiteboards and pens
- "linear" chocolate covered candy (e.g. Pocky)

0.0.5 Wrapup

Emphasize that students must express each integrand in terms of a single variable prior to integration.

Emphasize that each integral must be positive!

Discuss several different ways of doing this problem (see below).

0.1 Details

0.1.1 In the Classroom

- Make sure the shape of the pretzel is clear! It might be worth drawing it on the board.
- Some students will work geometrically, determining ds on each piece by inspection. This is fine, but encourage such students to try using $d\vec{r}$ afterwards.
- Polar coordinates are natural for all three parts of this problem, not just the circular arc.
- Many students will think that the integral "down" the y-axis should be negative. They will argue that $ds = dy$, but the limits are from 2 to 0. The resolution is that $ds = |dy\,\hat{\boldsymbol{x}}| = |dy| = -dy$ when integrating in this direction.
- Unlike work or circulation, the amount of chocolate does not depend on which way one integrates, so there is in fact no need to integrate "down" the y-axis at all.
- Some students may argue that $d\vec{\boldsymbol{r}} = \hat{\boldsymbol{T}}\,ds \Longrightarrow ds = d\vec{\boldsymbol{r}}\cdot\hat{\boldsymbol{T}}$, and use this to get the signs right. This is fine if it comes up, but the unit tangent vector $\hat{\boldsymbol{T}}$ is not a fundamental part of our approach.
- There is of course a symmetry argument which says that the two "legs" along the axes must have the same amount of chocolate — although some students will put a minus sign into this argument!

0.1.2 Subsidiary ideas

- $ds = |d\vec{\boldsymbol{r}}|$