Student handout

Vector Differential: Rectangular Coordinates:

Find the general form for $d\vec{r}$ in rectangular coordinates by determining $d\vec{r}$ along the specific paths in the figure below.

- Path 1:

  $d\vec{r} =$

- Path 2:

  $d\vec{r} =$

- Path 3:

  $d\vec{r} =$

If all three coordinates are allowed to change simultaneously, by an infinitesimal amount, we could write this $d\vec{r}$ for any path as:

$d\vec{r} =$

This is the general line element in rectangular coordinates.

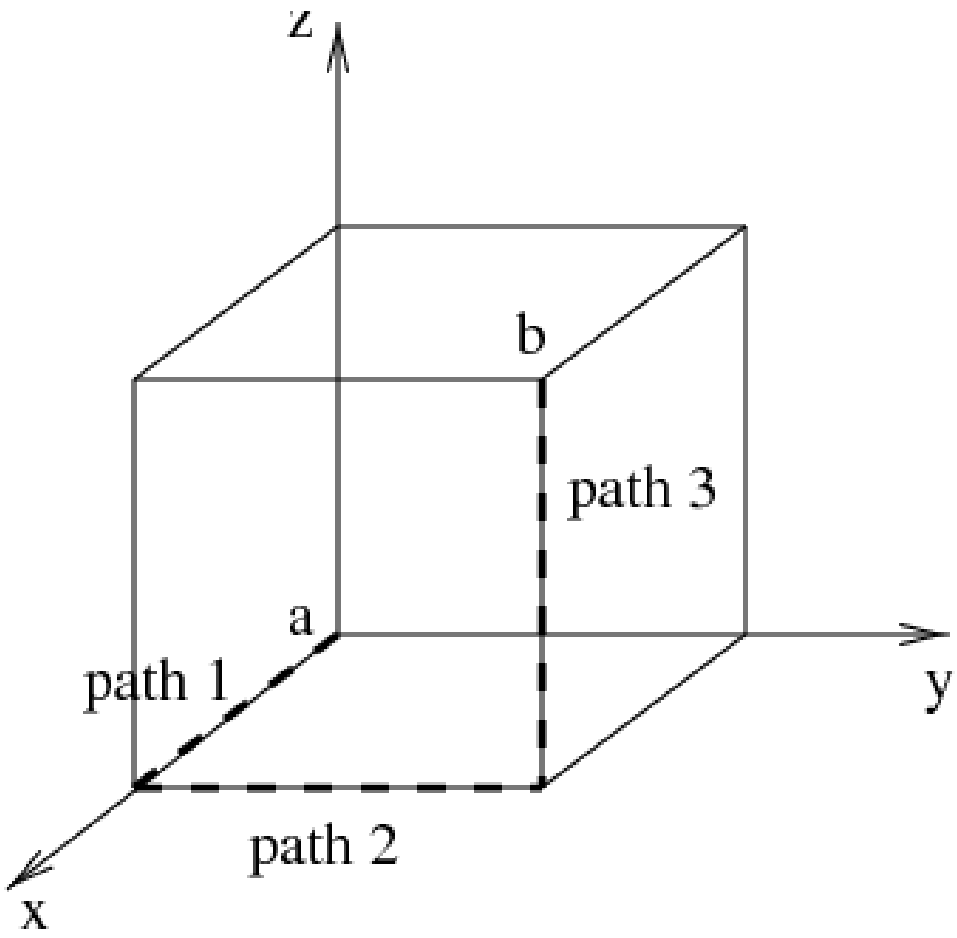


Figure 1: Figure 1: $d\vec{r}$ in rectangular coordinates

1 Instructor's Guide

1.1 Main Ideas

This activity allows students to derive formulas for $d\vec{r}$ in rectangular coordinates, using purely geometric reasoning. The formula forms the basis of a unified view of all of vector calculus, so this activity is essential. For more information on this unified view, see our publications, especially: Using differentials to bridge the vector calculus gap

1.2 Students' Task

Using a picture as a guide, students write down an algebraic expression for the vector differential in rectangular coordinates.

1.3 Introduction

Begin by drawing a curve (like a particle trajectory, but avoid "time" in the language) and an origin on the board. Show the position vector $\vec{r}$ that points from the origin to a point on the curve and the position vector $\vec{r} + d\vec{r}$ to a nearby point. Show the vector $d\vec{r}$ and explain that it is tangent to the curve.

We often do this rectangular case, paths 1-3, for the students, as a mini-lecture to get them started quickly before doing activity Vector Differential–Curvilinear.

1.4 Wrap-up

The only wrap-up needed is to make sure that all students have (and understand the geometry of!) the correct formulas for $d\vec{r}$.