Student handout

Gauss's Law:

$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{1}{\epsilon_0} Q_{\text{enc}}$$

Ampère's Law:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enc}}$$

Potentials:

$$\begin{aligned} \vec{E} &= -\vec{\nabla} V \\ \vec{B} &= \vec{\nabla} \times \vec{A} \end{aligned}$$

Maxwell's Equations (static, in vacuum):

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= 0 \\ \vec{\nabla} \times \vec{B} &= \mu_0 \vec{J} \end{aligned}$$

Superposition Laws:

$$\begin{aligned} V(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')\, d\tau'}{|\vec{r} - \vec{r}'|} \\ \vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')(\vec{r} - \vec{r}')\, d\tau'}{|\vec{r} - \vec{r}'|^3} \\ \vec{A}(\vec{r}) &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')\, d\tau'}{|\vec{r} - \vec{r}'|} \\ \vec{B}(\vec{r}) &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')\, d\tau'}{|\vec{r} - \vec{r}'|^3} \\ V(B) - V(A) &= -\int_A^B \vec{E} \cdot d\vec{r} \end{aligned}$$

Position Vectors

$$\begin{aligned}\vec{r} &= x\hat{x} + y\hat{y} + z\hat{z} \\ &= s\hat{s} + z\hat{z} \\ &= r\hat{r}\end{aligned}$$

The distance between two position vectors

1. In cylindrical coordinates:

$$|\vec{r} - \vec{r}'| = \sqrt{s^2 + s'^2 - 2s\,s'\cos(\phi - \phi') + (z - z')^2}$$

2. In spherical coordinates:

$$|\vec{r} - \vec{r}'| = \sqrt{r^2 + r'^2 - 2r\,r'\left[\sin\theta\sin\theta'\cos(\phi - \phi') + \cos\theta\cos\theta'\right]}$$

Area and Volume Elements:

$$\begin{aligned}d\vec{A} &= d\vec{r}_1 \times d\vec{r}_2 \\ d\tau &= \left(d\vec{r}_1 \times d\vec{r}_2\right) \cdot d\vec{r}_3 \\ d\tau &= s\,ds\,d\phi\,dz \\ d\tau &= r^2\,sin\theta\,dr\,d\theta\,d\phi\end{aligned}$$

Derivatives in Rectangular Coordinates:

$$\begin{aligned}\vec{\nabla} f &= \frac{\partial f}{\partial x}\,\hat{x} + \frac{\partial f}{\partial y}\,\hat{y} + \frac{\partial f}{\partial z}\,\hat{z} \\ \vec{\nabla}\cdot\vec{F} &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \\ \vec{\nabla}\times\vec{F} &= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}\end{aligned}$$

Derivatives in Cylindrical Coordinates:

$$\begin{aligned}\vec{\nabla} f &= \frac{\partial f}{\partial s}\,\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\,\hat{\phi} + \frac{\partial f}{\partial z}\,\hat{z} \\ \vec{\nabla}\cdot\vec{F} &= \frac{1}{s}\frac{\partial}{\partial s}\left(sF_s\right) + \frac{1}{s}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \\ \vec{\nabla}\times\vec{F} &= \left(\frac{1}{s}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial F_s}{\partial z} - \frac{\partial F_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial}{\partial s}\left(sF_\phi\right) - \frac{\partial F_s}{\partial \phi}\right)\hat{z}\end{aligned}$$

Derivatives in Spherical Coordinates:

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\,\hat{\phi} \\
\vec{\nabla}\cdot\vec{F} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi} \\
\vec{\nabla}\times\vec{F} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\left(\sin\theta F_\phi\right) - \frac{\partial F_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}\left(rF_\phi\right)\right)\hat{\theta} \\
&\quad + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rF_\theta\right) - \frac{\partial F_r}{\partial \theta}\right)\hat{\phi}
\end{aligned}$$

Lorentz Force Law:

$$\vec{F} = q_{\text{test}}\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

Step and Delta Functions:

$$\begin{aligned}
\frac{d}{dx}\theta(x-a) &= \delta(x-a) \\
\int_{-\infty}^{\infty} f(x)\delta(x-a)\,dx &= f(a)
\end{aligned}$$

Vector Calculus Theorems:

$$\begin{aligned}
\oint \vec{F}\cdot d\vec{A} &= \int \vec{\nabla}\cdot\vec{F}\,d\tau \\
\oint \vec{F}\cdot d\vec{\ell} &= \int (\vec{\nabla}\times\vec{F})\cdot d\vec{A}
\end{aligned}$$

Total Charge and Current:

$$\begin{aligned}
Q &= \int \rho(\vec{r}')d\tau' \\
I &= \int \vec{J}(\vec{r}')\cdot d\vec{A}'
\end{aligned}$$