Your group has a plastic surface and a contour map that represent the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. Solve the following problems together and discuss the results.

Instructor's guide

You may need to remind students that the gravitational potential energy is zero infinitely far away from Earth.

Compare Potentials: Rank the three points marked on the surface by gravitational potential energy from highest to lowest.

Instructor's guide

Answer: This answer is relatively intuitive using the surface, especially if students have recently completed the *Gravitational Potential Energy* activity. The higher a point, the higher the gravitational potential energy.

Identify Forces: What direction is the gravitational force at each of the marked points? Indicate the direction of each force with a vector on the contour map.

Instructor's guide

Hint: Students can rely on their intuition that the gravitational force points toward Earth.

Potential Pitfall: It is common for students to think that the force vectors are tangent to the surface. In fact, all three points exist in a horizontal plane, and their force vectors also only have horizontal components.

Extension: If you were to draw the vectors on the surface, how would those vectors be the same or different from the vectors on the contour map?

Rank the three points by the magnitude of the gravitational force.

Instructor's guide

As with the previous questions, this question is relatively intuitive.

Plot: Sketch a graph of the gravitational force *vs.* distance from the center of the Earth. Use the convention that positive forces point *away* from the center of the Earth.

Instructor's guide

Goal: Students to visualize the function $F(r)$.

Note: If students make the force positive, they are plotting the magnitude of the force. The component of the force is negative since it points opposite to increasing $\boldsymbol{r}$.

Examine Changes: At each point, imagine that the space station moves a small distance (about 10 m) directly toward the center of the Earth.

1. At each point, is the resulting *change in gravitational potential energy* positive, negative, or zero?

2. Rank the three points by the *magnitude* of the change in gravitational potential energy.

Instructor's guide

This question can be less intuitive than those above. Pay close attention to students' answers, as the conclusion in this question (that the change is larger closer to the Earth) is used in the conclusion of the activity.

Relate the Surfaces: The yellow surface represents the gravitational potential energy of an object close to the surface of Earth. How it is possible for both surfaces to correctly represent the gravitational potential energy when the object is near the Earth's surface?

Instructor's guide

This question is intended to get students to recognize that the force near the surface of the Earth, which is uniform, is simply a linear approximation to the actual force, which varies with distance. This idea can be especially difficult for students, but is a valuable connection to systems that students have already studied.

Generate Graphs: Draw graphs of gravitational potential energy vs. distance and gravitational force vs. distance for the yellow surface.

Find Patterns: Examine the graphs of gravitational potential energy and force for both the green and yellow surfaces.

What patterns do you see between gravitational force and gravitational potential energy?

Instructor's guide

The big takeaway of the activity is that gravitational force is related to the change (derivative) in the potential energy, **not** to the *value* of the potential energy. This can be an especially persistent idea because the graphs of U and F for universal gravitation are hard to distinguish for the green surface. The difference is much more clear for the yellow surface.

Instructor's guide

SUMMARY PAGE

What Students Learn:

- Recall intuitions about gravitational force and potential energy.
- The magnitude of the gravitational force is given by the derivative of (the change in) the gravitational potential energy.
- The direction of the gravitational force is opposite the sign of the slope of the potential energy.

Time Estimate: 20 minutes

Equipment

- Green "sphere" surface
- Dry-erase markers & erasers
- Whiteboard for each group
- Student worksheet for each student
- Contour map for each group

Introduction

- Students should be familiar with gravitational force and potential energy near the surface of the Earth.
- It can be helpful for students to be familiar with equations and graphs of universal gravitation, but it is not essential.
- Optional: We find it useful for the instructor to explain the context at the start of the activity.
- The surface of the Earth is marked by an indent in the surface near the lowest corner. The height of the surface corresponds to the value of gravitational potential energy.

Whole Class Discussion / Wrap Up:

- This activity is especially effective when given following the *Gravitational Potential Energy* activity.
- If students have done both activities, they will have graphs of both $U(r)$ and $F(r)$ (if not, you might encourage students to sketch $U(r)$ as well). Ask students to compare the two graphs directly, and especially to think about how the graphs can help them identify patterns. Many students will think that because the graphs look nearly identical, U and F must be directly proportional.
- The meaning of the negative sign for U and F is especially challenging. Asking students to describe the meaning of the negative sign in each case (it indicates the *value* of U but the *direction* of F) and calling attention to the difference can help them develop more sophisticated reasoning about negative signs.