

The Sign of Work: For the force vector shown above, draw three paths such that the work done by $\vec{F}$ is positive, zero, and negative for small displacements along each path.

Instructor's guide

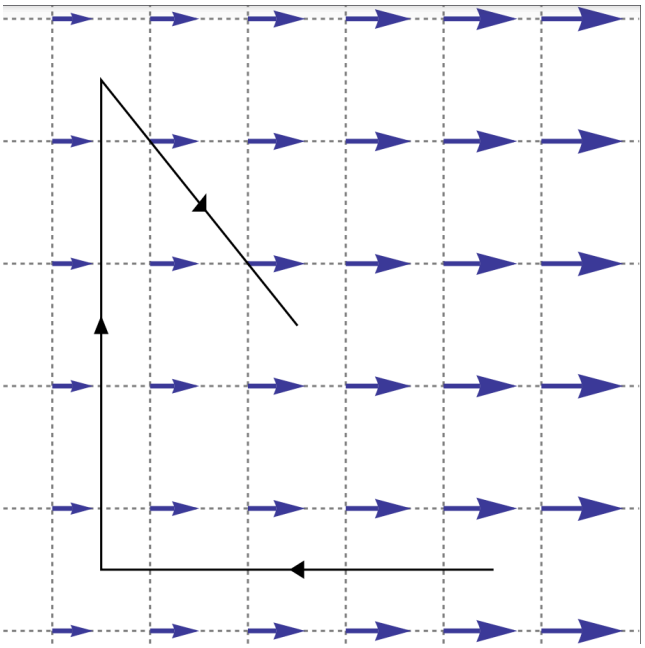
Goal: This gets at the dot product relationship students will need to perform the vector path integral. It does not get at the ideas that you have to chop the whole path into small pieces to do the integral (see next question).

Follow-Up: "Is there only one correct path for each?" Students usually draw the parallel/antiparallel/perpendicu paths and do not consider other options.

Estimate Vector Line Integral: For each segment of the path on the vector field $\vec{F}$ shown below, estimate the value of the integral:

$$\int_{\text{path}} \vec{F} \cdot d\vec{r}$$

where the side of each square is 1 cm and the length of the longest arrow is 10 units (appropriate for the field $\vec{F}$).



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Discussion: Units Students might ask about the units of $\vec{F}$. This is a great place to talk about how for an integral, the units of the result are the units of the integrand times the units of the infinitesimal (students often ignore the units of the infinitesimal). Also, for E&M, students are going to do a similar integral with the electric field - which does not have units of force - so being mindful about the units of the field is important.

Answer: We particularly recommend students use the *Chop, Multiply, Add* strategy, but there are other reasonable methods here.

For the leftward path, students need to be able to chop the path into segments (we recommend 4 segments), find the value of the dot product for that path, and then add up those four values to get the line integral.

For the upward path, the path is always perpendicular to the vector field, so the line integral is zero.

For the diagonal path (the hardest path), students need to chop up the path and consider the different magnitude at each point, but they also need to account for the nontrivial angle between the path and the vector field. This can be done in several ways—students will often try to find an explicit form for $\vec{dr}$, but they can also use a geometric argument to use only the vertical component of the path.

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SUMMARY PAGE

What Students Learn:

- Recall the relationship between the sign of the dot product and the orientation of the vectors.
- Use graphical methods to estimate the value of a vector line integral.
- Lay the groundwork for thinking about conservative and non-conservative vector fields.

Time Estimate: 15 minutes

Equipment

- Dry-erase markers & erasers
- Whiteboard for each group
- Student handout for each student

Introduction

- This activity is a warmup activity and students do not need an introduction.

Whole Class Discussion / Wrap Up:

- This activity is a warmup to two other activities: *Number of Paths* and *Work by an Electric Field*. We highly recommend doing this activity before either of these activities.
- Because there are multiple possible answers for the first part of the activity, and multiple possible strategies for evaluating the integral in the second part, this activity is particularly well suited to a whole-class discussion in which different groups of students are asked to share their answers and discuss the advantages and disadvantages of each.