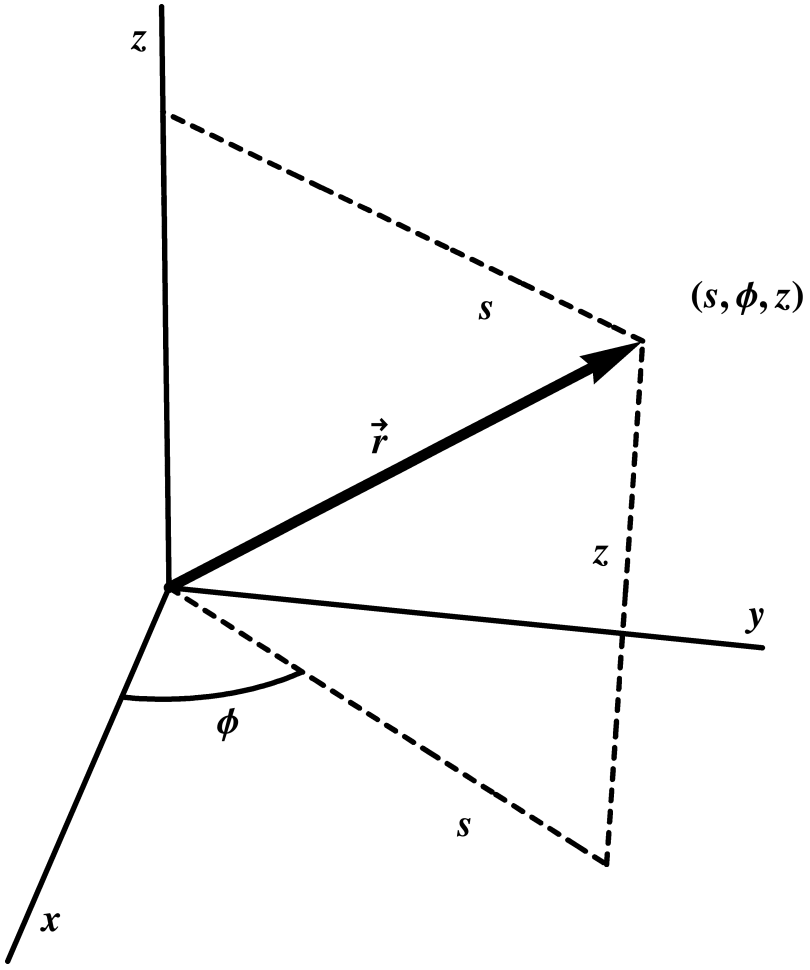
Student handout

1 Cylindrical Coordinates

For the cylindrical coordinate system shown below, draw three surfaces: one for constant s, one for constant ϕ, and one for constant z.

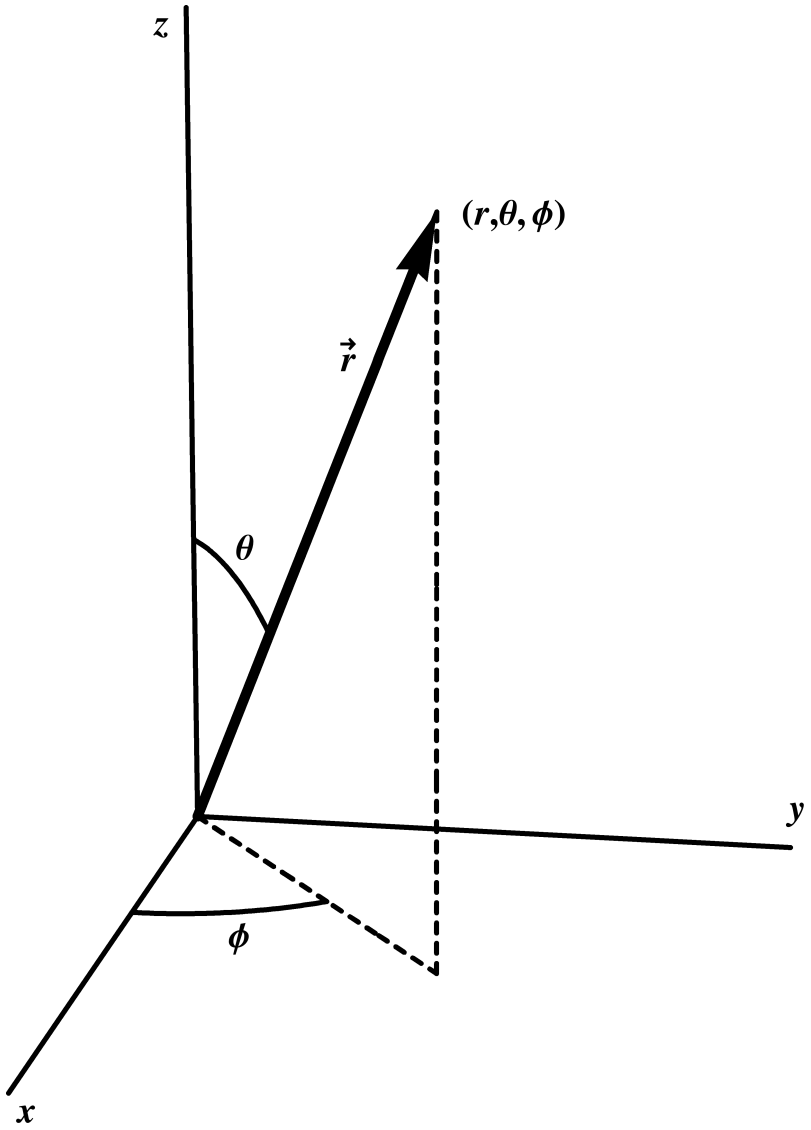


$$x = s\cos\phi \tag{1}$$
$$y = s\sin\phi \tag{2}$$
$$z = z \tag{3}$$

$$0 \leq s < \infty \tag{4}$$
$$0 \leq \phi < 2\pi \tag{5}$$
$$-\infty < z < \infty \tag{6}$$

2 Spherical Coordinates

For the spherical coordinate system shown below, draw three surfaces: one for constant r, one for constant θ, and one for constant ϕ.



$$x = r\ \sin\theta\ \cos\phi \tag{7}$$
$$y = r\ \sin\theta\ \sin\phi \tag{8}$$
$$z = r\ \cos\theta \tag{9}$$

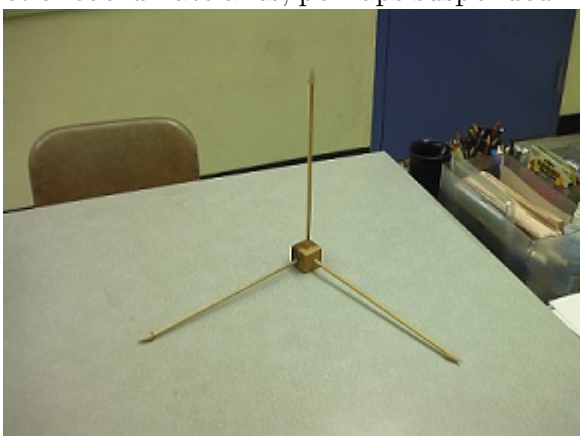
$$0 \leq r < \infty \tag{10}$$
$$0 \leq \theta < \pi \tag{11}$$
$$0 \leq \phi < 2\pi \tag{12}$$
$$\tag{13}$$

3 Instructor's Guide

3.1 Introduction

First, show students diagrams of cylindrical and spherical coordinates. Discuss common notation systems, especially that mathematicians and physicists use opposite notations for the angles θ and ϕ. Don't forget to discuss the ranges of each of the coordinates.

It can be very helpful to have a set of coordinate axes, perhaps suspended from the ceiling somewhere in the room, to refer to as needed.





Attach a string to the z-axis at the origin so that it can move freely and demonstrate, by moving the string appropriately, how the various angles in cylindrical and spherical coordinates change.

3.2 Student Conversations

Ask students to check their understanding by sketching several coordinate equals constant surfaces on their small whiteboards. Appropriate figures are attached. Beware, this part of the activity can take longer than you expect; you can try to speed things up by limiting the coordinate equals constant sketches to s=constant and ϕ=constant in cylindrical and θ=constant in spherical.

- When sketching s = constant in cylindrical coordinates, some students sketch a sphere.
- When sketching ϕ = constant in cylindrical (or spherical) coordinates, many students are puzzled about why the answer is a half-plane instead of a whole plane. This is a good opportunity to emphasize the ranges for the various variables.

– Cylindrical Coordinates:

$$0 \leq s < \infty \tag{14}$$
$$0 \leq \phi < 2\pi \tag{15}$$
$$-\infty < z < \infty \tag{16}$$

– Spherical Coordinates:

$$0 \leq r < \infty \tag{17}$$
$$0 \leq \theta < \pi \tag{18}$$
$$0 \leq \phi < 2\pi \tag{19}$$
$$\tag{20}$$

- When sketching $\theta =$ constant in spherical coordinates, most students do not understand that the answer is a cone.