1 Instructor's Guide

1.1 Introduction

As described here, this activity has been done as a whole class activity where students answer a series of small whiteboard questions (SWBQ) as the instructor makes cuts in a pumpkin to construct a volume element in spherical coordinates. This activity can be done as a small group activity by giving pieces of pineapple slices and pumpkins, together with appropriate children's pumpkin cutting tools, while students are doing Scalar Surface and Volume Elements or Vector Surface and Volume Elements.

If done as an instructor led whole class activity, the following provides a structure for the activity done in spherical coordinates using a pumpkin.

- Let's define a set of coordinates $\hat{x}$ and $\hat{y}$ ($\hat{z}$ is through the pumpkin stem). I'm going to start by drawing a line around the equator to help my drawing.
  - SWBQ: What is the circumference of this circle? Answer: $l = 2\pi r$
- Now I'm going to pick a vector $\vec{r}$, which we will examine. When drawing figures (or pumpkins), I recommend that you never pick an angle that is close to 90° or 45°, since that makes it way easier to distinguish between θ and $\frac{\pi}{2} - \theta$.
- Draw a longitude line.
  - SWBQ: What is the arc length from $\vec{r}$ to the stem? Answer: $l = r\theta$
  - SWBQ: What is the distance along the equator from $\hat{x}$? Answer: $r\phi$
- Now let's consider an infinitesimal volume of pumpkin. Pro tip: you always want to draw infinitesimal quantities as medium-sized, otherwise your drawing will be illegible, and won't actually help you. In this case, I'll pick a $d\theta$ and a $d\phi$.
  - SWBQ: What is this small distance along the equator? Answer: $dl = rd\phi$
  - SWBQ: What is this small distance along the longitude? Answer: $dl = rd\theta$
- Draw a latitude circle at $\vec{r}$.
  - SWBQ: What is the circumference of this circle? Answer: $l = 2\pi s = 2\pi r \sin\theta$
- Cut out the pumpkin chunk.
  - SWBQ: What is the width of my pumpkin chunk? Answer: $dl = sd\phi = r\sin\theta d\phi$
- So when we put these distances together (including the thickness of the chunk dr), we find that the volume of our pumpkin chunk is $d\tau = dr(rd\theta)(r\sin\theta d\phi) = r^2 \sin\theta dr d\phi d\theta$